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Uncovered interest parity and threshold cointegration approach: theory and evidence

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**Uncovered interest parity and threshold cointegration approach:
theory and evidence**

by

Seunghwan Kim

**A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY**

Major: Economics

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2002

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Seunghwan Kim
has met the dissertation requirements of Iowa State University**

Signature was redacted for privacy.

Major Professor

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For the Major Program

To my wife, Hyun-ok Lee

TABLE OF CONTENTS

ABSTRACT	v
CHAPTER 1. INTRODUCTION	1
CHAPTER 2. UNCOVERED INTEREST PARITY (UIP)	5
CHAPTER 3. LITERATURE REVIEW ON UIP	8
1. Early studies on UIP	8
2. Theoretical and empirical studies on the UIP puzzle	10
1) The forward risk premium approach	12
2) The irrational forecasting behavior approach	15
3) The “peso problem” approach	17
4) The policy intervention approach	20
CHAPTER 4. UIP AND THRESHOLD COINTEGRATION	25
1. Long-run equilibrium relationship and threshold cointegration	25
2. Literature review on the application of threshold cointegration	27
3. Methodology	31
4. Empirical Analysis	38
1) Exchange rate data	38
2) Conventional UIP test	44
3) Linear cointegration tests	45
4) Threshold nonlinearity tests	52
5) Model specification test	58
6) Model estimation and evaluation	60
7) Out-of-sample Forecasting	69
CHAPTER 5. CONCLUSION	72
REFERENCES CITED	75

ABSTRACT

Although the uncovered interest parity (UIP) condition has played an important role in many theoretical and empirical models of open-economy macroeconomics, the conventional empirical test for the validity of UIP has shown that the null hypothesis of the UIP condition is almost always rejected and, especially, the slope estimate of the forward premium is significantly negative. Four different approaches to explaining this UIP puzzle have been introduced so far, but none of them has succeeded in providing a fully acceptable rationale and empirical test result. The present paper investigates the UIP puzzle using the threshold cointegration approach for major four currencies: the Canadian dollar, the Japanese yen, the German mark, and the British pound. We find that the slope estimate of the forward premium in the context of the threshold vector error-correction model (TVECM) has a positive or negative sign, depending on currencies. Based on this finding, we conclude that the threshold cointegration approach does not provide robust evidence for the UIP condition, and that the UIP puzzle remains partially unsolved. However, our paper gives some contributions to the study of the UIP puzzle and the application of the threshold cointegration approach. First, we provide a general review of the theoretical and empirical studies on the UIP condition including the threshold cointegration approach. Second, we find that the spot and forward exchange rates for the four major currencies have a bivariate threshold cointegration property. Third, we estimated the band TVECM for the spot and forward exchange rates of these currencies. Forth, we constructed out-of-sample forecasts using the TVECM and four alternative models, and found that the TVECM has the best forecasting ability based on root-mean-square-error (RMSE) and mean-absolute-error (MAE) criteria. According to this finding, the estimated TVECM can be used as a predictor of short-term movements in exchange rates although the estimated results are inconsistent with the UIP condition.

CHAPTER 1. INTRODUCTION

The hypothesis of uncovered interest parity (hereafter UIP) has played an important role in understanding exchange rate movements, especially in Dornbusch's (1976) extension of the Mundell-Flemming model. It has also been a cornerstone of many theoretical and empirical models of open-economy macroeconomics. Without UIP, these models lack an essential element so that their results might be misleading. As a result, the validity of UIP has been one of the most challenging topics of international economics over the last thirty years.

Simply speaking, UIP implies that the expected future change in the spot exchange rate must be fully determined by the interest rate differential (or the forward premium) between two countries under three basic assumptions: no default risk in both domestic and foreign assets, market participants' risk neutrality and rational expectations.¹ The large literature on UIP, however, indicates that the exchange rate change does not have one-to-one relationship with the interest differential (or the forward premium) and, furthermore, these two variables are negatively correlated with each other contrary to that implied by UIP. In a survey of 75 published estimates of the slope coefficient from the regression of the future change in the spot exchange rate on the current forward premium, Froot (1990) found that the average point estimate of the slope coefficient is -0.88 and that only a few of the estimates have the positive sign predicted by UIP.

Early studies on UIP attributed this contradiction to the existence of forward risk premia in foreign exchange markets, relaxing the assumption of risk neutrality. Fama (1984) and Bilson (1981, 1985) investigated the UIP puzzle and interpreted it as evidence of risk

¹ The forward premium is the percentage difference between the current forward and spot exchange rates.

premia on holding foreign exchange. Domowitz and Hakkio (1985), Diebold and Pauly (1988) and Koning and Straetmans (1997) considered time-varying forward risk premia in the context of the autoregressive conditional heteroskedasticity (ARCH) model and the structural heteroskedasticity model as the cause of the UIP puzzle. On the other hand, some studies took a different path, explaining the UIP puzzle as a violation of rational expectations. Using market survey data, Frankel and Froot (1987) and Froot and Frankel (1989) presented evidence that market participants' expectations could be irrational. Their approach, however, has not provided a plausible rationale for the market participants' systematic irrationality. Lewis (1994) and Evans and Lewis (1995) seek to explain the systematic forward exchange rate forecast errors without violating the rational expectation assumption. Their approach shows that the estimate of the slope coefficient could be downward biased as the result of a "peso problem" effect or rational learning. McCallum (1994) suggested that even if the UIP condition holds, the slope coefficient could be less than unity as long as policy interventions create frictions in the foreign exchange market. Meredith and Chinn (1998) attempted to generalize McCallum's idea by incorporating output and inflation into a policy response function and testing the UIP condition with considerably longer maturity data. However, none of these approaches has provided a satisfactory explanation of the UIP puzzle.

This paper takes a completely different approach from the previous rationales for the UIP puzzle. The main idea of this paper is that the UIP puzzle is due to a failure to adequately model the stochastic features of the data generating process as well as a failure to give suitable theoretical rationales. The threshold cointegration model will be introduced and estimated to model the particular time series behavior of spot and forward exchange rates. In

a conventional cointegration (or an error correction) model, the adjustment process of a deviation to the long-run equilibrium relationship is modeled as being linear. However, when there are market frictions or discrete policy responses, no adjustment process from a deviation to the long-run equilibrium takes place until the deviation reaches a certain threshold. In other words, the recovery process to the long-run equilibrium begins to work only if the deviation is so large as to exceed the threshold. Since Balke and Fomby (1997) proposed a threshold cointegration approach, many authors have applied this method to various topics. Balke and Fomby themselves examined the relationship between the Fed Funds rate and the Discount rate. Michael, Nobay and Peel (1997) used threshold cointegration to investigate purchasing power parity (PPP) in the presence of transaction costs. Weidmann (1997) used it to study the long-run Fisher effect under market intervention by a central bank. Balke and Wohar (1997) applied the approach to covered interest parity (CIP). Coakley and Fuertes (1999) used this method to test the forward rate unbiased hypothesis (FRUH).

In this paper, we will apply a band threshold vector error correction model (TVECM) in an effort to capture a threshold cointegration relationship between spot and forward exchange rates. Then, we will examine the validity of the band TVECM as an explanation of the UIP puzzle. The data set that will be used in this paper are 4 major currencies whose values are measured against the U.S. dollar: the Canadian dollar, the Japanese yen, the German mark, and the British pound.

The contents of this paper are organized as follows. In chapter 2, we elaborate on the concepts of UIP and covered interest parity (CIP). Chapter 3 provides a more detailed review of the theoretical and empirical literature on UIP. In chapter 4, we attempt to test UIP

by a threshold cointegration approach that is clearly different from previous approaches to the UIP puzzle. The band TVECM is introduced and estimated. Chapter 5 concludes.

CHAPTER 2. UNCOVERED INTEREST PARITY (UIP)

To explain UIP, it is easier to begin with covered interest parity (hereafter CIP). Suppose that both domestic and foreign assets are free of default risk, and that the conditions for risk-free arbitrage are satisfied. Under these assumptions, foreign assets on which forward cover has been obtained are perfect substitutes for domestic assets. Then, arbitrage transactions bring the gross return on a unit of domestic currency invested in domestic assets into equality with the gross return from taking one unit of domestic currency and buying $1/S_t$ units of foreign currency, where S_t is the spot exchange rate, investing it in foreign assets, and selling the gross return forward at the forward exchange rate (F_t). Because this entire process can be conducted at time t , it involves no risk. Algebraically, CIP can be written as:

$$(1 + i_{t,k}) = (1 + i_{t,k}^*) \frac{F_{t,t+k}}{S_t} \quad (1)$$

where S_t and $F_{t,t+k}$ are, respectively, the spot and k -period-forward exchange rates in levels expressed as the price of foreign currency in terms of domestic currency at time t , and $i_{t,k}$ and $i_{t,k}^*$ are, respectively, the k -period net interest rates on domestic and foreign assets that are identical in all respects except for the currency of denomination at time t .

Taking the logarithm of both sides of equation (1) and using the approximation $\ln(1+i_t) \approx i_t$ gives the CIP condition:

$$f_{t,t+k} - s_t = i_{t,k} - i_{t,k}^* \quad (2)$$

where $f_{t,t+k}$ and s_t are the logarithmic expressions of $F_{t,t+k}$ and S_t .² The left-hand-side is called the forward premium (or forward discount) on the domestic currency.³ Many empirical studies on the major industrial countries have indicated that deviations from CIP probably lie within transaction costs.⁴

Substituting the expected future spot rate for $f_{t,t+k}$ in (2) gives the UIP condition:

$$E_t s_{t+k} - s_t = i_{t,k} - i_{t,k}^* \quad (3)$$

where E_t is the expectation operator and s_{t+k} is the log of the spot exchange rate at time $t+k$. The log of the expected future spot exchange rate $E_t s_{t+k}$ is not necessarily the log of the forward exchange rate $f_{t,t+k}$.⁵ The UIP condition expressed as equation (3) implies that the expected change in the spot exchange rate equals the current interest differential between two countries.⁶

Equation (3) is, however, not directly testable because the future spot exchange rate s_{t+k} is not known at time t . Hence, the measurement of market expectations of the future spot exchange rate $E_t s_{t+k}$ is very important for deciding the behavior of equation (3). If market participants form rational expectations of the future spot exchange rate, the future spot exchange rate equals the rationally expected value at time t plus a white noise error term at

² The logarithmic expression for the spot and forward rates are independent of the way exchange rates are quoted. If the rates are not expressed in logarithmic form, Siegel's paradox (1972) would result.

³ Many authors use the forward discount rather than the forward premium because it can be easily positive and negative.

⁴ See Branson (1969), Aliber (1973), Frenkel and Levich (1975, 1977) and Clinton (1988).

⁵ Hereafter, even though we do not use the expression of "log", the spot and forward exchange rate are always expressed in log form.

⁶ If we abandon the investors' risk neutrality assumption in UIP, the forward exchange rate can be expressed as the expected future spot rate plus a forward risk premium. Incorporating a forward risk premium into UIP gives:

$E_t s_{t+k} - s_t = i_{t,k} - i_{t,k}^* - \pi_{t,k}$, where $\pi_{t,k}$ is a forward risk premium to compensate for the risk of holding foreign assets over domestic assets.

time $t+k$, ε_{t+k} , that is orthogonal to all information available at time t .⁷ This result comes from the property of rational expectation forecast errors that $E[\varepsilon_{t+k} | \Omega_t] = 0$, where $E[\cdot | \Omega_t]$ denotes the mathematical expectation conditional on all information available at time t . We can express this idea as the following simple equation:

$$E_t^R s_{t+k} = s_{t+k} - \varepsilon_{t+k} \quad (4)$$

where E_t^R denotes the rational expectation. Substituting equation (4) into (3) and rearranging yields:

$$s_{t+k} - s_t = (i_{t,k} - i_{t,k}^*) + \varepsilon_{t+k} \quad (5)$$

In equation (5), all variables except the disturbance term are observable. Furthermore, the disturbance term is uncorrelated with $i_{t,k} - i_{t,k}^*$. So, we are able to test the equation (5) using the following regression equation:

$$\Delta s_{t,t+k} = \alpha + \beta(i_{t,k} - i_{t,k}^*) + u_{t+k} \quad (6)$$

where Δ is the backward difference operator and u_{t+k} is an error term that has conditional expectation zero. If we assume that u_{t+k} is uncorrelated with the interest differential, we can estimate β using ordinary least squares (OLS) and apply standard t-tests and F-tests to test the joint null hypothesis $H_0: \alpha=0, \beta=1$ and the error term has a conditional mean of zero.⁸

⁷ The error term can be allowed to follow a martingale difference process, which encompasses a sequence of errors with different variances.

⁸ Non-zero values of the constant term can be accepted under UIP. Meredith and Chinn (1998) give three reasons. First, Jensen's inequality implies that the expectation of a ratio is not the same as the ratio of

CHAPTER 3. LITERATURE REVIEW ON UIP

1. *Early studies on UIP*

In practice, the forward premium (or discount) often substitutes for the interest differential in equation (6) because CIP is widely believed to hold. Hence, the early studies presented below will actually be based on the forward premium (or discount) instead of the interest differential.

Fama (1984) regresses the change in the spot exchange rate on the forward premium:

$$\Delta s_{t+1} = \alpha + \beta (f_t - s_t) + u_{t+1} \quad (7)$$

where Δ is the backward difference operator and u_{t+1} is an error term that has conditional expectation zero relative to period t information. If the forward premium is an unbiased predictor of the future change in the spot exchange rate, then β must be unity in (7). The results of this regression using the dollar exchange rate against the Canadian dollar, the German mark, the British pound and the Japanese yen over the floating exchange rate period from 1973 to 1982 show that the estimates of β are -0.87 , -1.32 , -0.90 and -0.29 , respectively, which are all significantly negative and, therefore, significantly different from unity.⁹ Levich (1979), Bilson (1981, 1985), Longworth (1981), and Huang (1989) provide

expectations. Second, the constant term may reflect a constant exchange forward risk premium if we relax the risk neutrality assumption. Third, the constant term may include the default risk of holding assets.

⁹ Fama (1984) originally used the data of 9 countries (Belgium, Canada, France, Italy, Japan, Netherlands, Switzerland, United Kingdom, West Germany). The estimates of the remaining currencies are -1.58 (Belgium), -0.87 (France), -0.51 (Italy), -1.43 (Netherlands) and -1.14 (Switzerland).

similar results over other sample periods and currencies.¹⁰ This result is often referred to as the forward discount anomaly, the forward discount bias or the forward discount puzzle.¹¹

Others have regressed the spot exchange rate on the lagged forward rate:¹²

$$s_{t+1} = \alpha + \beta f_t + u_{t+1} \quad (8)$$

In the above equation, the UIP condition is rejected if β is significantly different from unity and/or the error term is not white noise. This equation implies that if the foreign exchange market is efficient, the forward exchange rate should be an unbiased estimator of the future spot exchange rate.¹³ This argument is also called the forward rate unbiased hypothesis (FRUH). Most empirical results from this regression show that the OLS estimate of β is close to unity.

The OLS estimates of β over equations (7) and (8) are strikingly different in both magnitude and sign, even for the same data and sample period. One explanation provided by Taylor (1995) is that when equation (7) is the true model, the OLS estimator of the slope coefficient in regression equation (8) is spuriously driven to one. In particular, equation (7) can be reparameterized as:

$$s_{t+1} = \alpha + \beta f_t + [(1 - \beta)s_t + u_{t+1}] \quad (9)$$

¹⁰ Some authors have regressed the forecast error instead of the spot exchange rate change on the forward premium: $s_{t+1} - f_t = \alpha + \gamma(f_t - s_t) + u_{t+1}$. This equation is the same as equation (7) with $\gamma = \beta - 1$.

¹¹ However, McCallum (1994) finds that when estimates are obtained for equation (7) with Δs_{t-j} for $j=2,3, \dots$ instead of $j=1$, the slope coefficient estimates revert to unity, similar to those for equation (8).

¹² See Frenkel (1976), Levich (1979), Frenkel (1981), Bilson (1981), Longworth (1981), Edwards (1983), and Chiang (1986, 1988).

¹³ The other popular method of testing UIP (or the forward rate unbiasedness hypothesis), based on equation (8), is $\Delta s_{t+1} = \alpha + \beta \Delta f_t + u_{t+1}$.

The ordinary least square (OLS) estimates in equation (9) are obtained through minimizing the sum of squared residuals which is minimized when the estimate of β equals unity. This intuition implies that the OLS method will drive the β estimate in equation (9) toward unity regardless of the true value of β .

If both s_t and f_t are nonstationary then the standard sampling theory is not applicable and standard hypotheses tests are not reliable any more in the regression of s_{t+1} on f_t . If s_t and f_t are nonstationary and not cointegrated then equation (7) involves a regression of a stationary variable on a nonstationary variable (and a nonstationary error term), which is also a nonstandard regression. However, the nonstationarity of s_t and f_t does not fully rationalize the difference between the estimates of β obtained from equations (7) and (8). If s_t and f_t are nonstationary but cointegrated, then the OLS estimator of β in (8) is superconsistent and the estimator of β in (7) is consistent.

2. Theoretical and empirical studies on the UIP puzzle

The fact that the OLS estimates of β using equation (7) are inconsistent with theoretical values under UIP requires us to give plausible explanations for the puzzle. Investigating the relevant literature on the UIP puzzle provides at least four clearly different types of approach. Before we begin to explore each approach in more detail, let us provide clear definitions of some language that will be used often in the later section of this paper. The forward risk premium ($f_t - E_t s_{t+1}$) is the log of the forward exchange rate minus the log of the expected future spot exchange rate. The expectational error ($E_t s_{t+1} - s_{t+1}$) is the expected

log of the future spot exchange rate minus the realized log of the future spot exchange rate. The forward exchange rate forecast error ($f_t - s_{t+1}$) is the log of the forward exchange rate minus the log of the spot exchange rate. Under risk neutrality and the rational expectations assumption, the forward risk premium becomes zero and expectational errors become a white noise process. Figure 1 shows the relationship among these three variables.

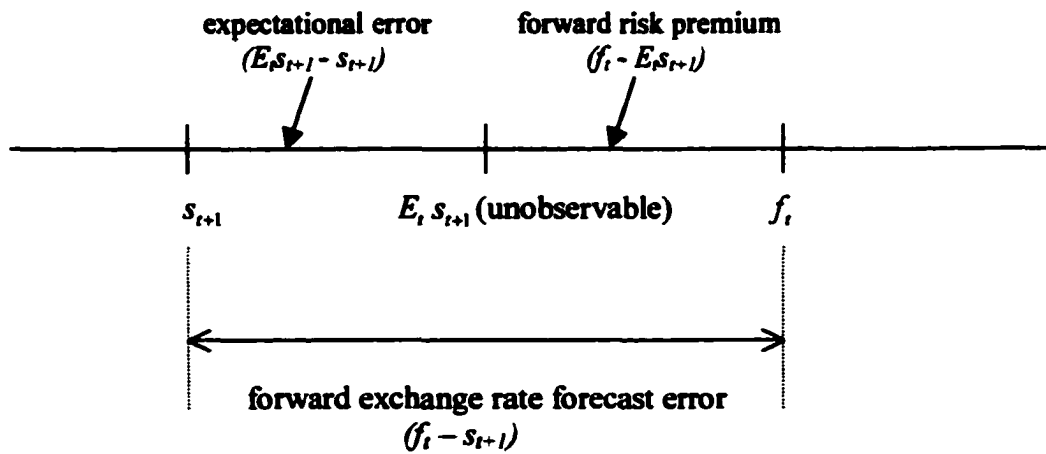


Figure 1. The relationship among s_{t+1} , $E_t s_{t+1}$ and f_t .

The first approach to the explanation of the UIP puzzle is that the deviation of the slope estimate from unity is due to a forward risk premium to compensate for holding foreign assets relative to domestic assets. The second approach considered is that the market participants' expectation for the future spot rate is irrational and thus the expectational errors ($E_t s_{t+1} - s_{t+1}$) in the spot exchange rate are systematically correlated with the forward premium or the interest differential.¹⁴ The third approach is the so-called "peso problem"

¹⁴ This argument implies that $E_t s_{t+1} \neq E_t^R s_{t+1} = s_{t+1} - \varepsilon_{t+1}$, where E_t^R denotes the rational expectation and ε_{t+1} is uncorrelated with all information available at time $t+1$, and that $\text{cov}(E_t s_{t+1} - s_{t+1}, i_t - i_t^*) \neq 0$ or $\text{cov}(E_t s_{t+1} - s_{t+1}, f_t - s_t) \neq 0$.

effect which refers to the possibility that the market participants' expectations reflect a major policy shift that does not occur in the sample period. Finally, the joint consideration of a monetary policy response function with UIP enables us to explain an OLS estimate that is much different from unity. We will examine each approach in detail in the following sections.

1) The forward risk premium approach

Fama (1984) argued that the negative deviation from unity of the slope coefficient in equation (7) is possible if the forward risk premium on holding foreign exchange is extremely high and volatile. Adding a forward risk premium, π_t , and assuming rational expectations, we obtain the following representation of UIP:

$$s_{t+1} - s_t = f_t - s_t - \pi_t + \varepsilon_{t+1} \quad (10)$$

where ε_{t+1} is a white noise forecast error. We can test equation (10) using the following regression equation:

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t) - \gamma\pi_t + u_{t+1} \quad (11)$$

The OLS estimator of β is:¹⁵

$$\hat{\beta} = \frac{\text{cov}(f_t - s_t, s_{t+1} - s_t)}{\text{var}(f_t - s_t)} = 1 - \beta_{\pi}, \quad \text{where } \beta_{\pi} = \frac{\text{cov}(E_t s_{t+1} - s_t, \pi_t) + \text{var}(\pi_t)}{\text{var}(f_t - s_t)} \quad (12)$$

¹⁵ The numerator of the below equation can be expanded as follows:

$$\begin{aligned} \text{cov}(f_t - s_t, s_{t+1} - s_t) &= \text{cov}(f_t - s_t, f_t - s_t - \pi_t + \varepsilon_t) = \text{var}(f_t - s_t) - \text{cov}(f_t - s_t, \pi_t) \\ &= \text{var}(f_t - s_t) - \text{cov}(E_t s_{t+1} - s_t + \pi_t, \pi_t) = \text{var}(f_t - s_t) - \text{cov}(E_t s_{t+1} - s_t, \pi_t) - \text{var}(\pi_t) \end{aligned}$$

If equation (11) represents the true model and the forward risk premium is correlated with the forward premium, omission of the forward risk premium term induces an omitted variable bias (β_p) in the OLS estimate of β .

For the estimate of β to be below 0.5 in a large sample, the forward risk premium must be more volatile than the expected change in the future spot exchange rate, i.e., $Var(\pi_t) > Var(E_t s_{t+1} - s_t)$.¹⁶ In addition, if the estimate of β in equation (7) is to be negative in a large sample, the covariance between the forward risk premium and the expected change in the spot exchange rate must be negative, i.e., $Cov(E_t s_{t+1} - s_t, \pi_t) < 0$.¹⁷ According to Lewis (1994), however, neither static nor general equilibrium models can explain Fama's condition that the forward risk premium must be more volatile than the expected change in the spot exchange rate.

Because of the theoretical relationship between risk and the second moments of asset price distributions, researchers have often modeled the forward risk premium as a function of the variance of either the forecast errors or of the exchange rate movements. This is called a forward risk premium model with conditional heteroscedasticity. Domowitz and Hakkio (1985) and Diebold and Pauly (1987) estimated the UIP relationship within an autoregressive conditional heteroskedastisty (ARCH) framework, which is described in the following equations.

¹⁶ If $\hat{\beta} < 1/2$, then $\frac{1}{2} \text{var}(f_t - s_t) < \text{cov}(E_t s_{t+1} - s_t, \pi_t) + \text{var}(\pi_t)$.

Since $\text{var}(f_t - s_t) = \text{var}(\pi_t) + 2 \text{cov}(E_t s_{t+1} - s_t, \pi_t) + \text{var}(E_t s_{t+1} - s_t)$, substituting into the above equation yields $\text{var}(\pi_t) > \text{var}(E_t s_{t+1} - s_t)$.

¹⁷ If $\hat{\beta} < 0$, then $\text{var}(f_t - s_t) < \text{cov}(E_t s_{t+1} - s_t, \pi_t) + \text{var}(\pi_t)$.

Since $\text{var}(f_t - s_t) = \text{var}(\pi_t) + 2 \text{cov}(E_t s_{t+1} - s_t, \pi_t) + \text{var}(E_t s_{t+1} - s_t)$, substituting into the above equation yields $\text{cov}(E_t s_{t+1} - s_t) + \text{var}(E_t s_{t+1} - s_t) < 0$ which implies $\text{cov}(E_t s_{t+1} - s_t, \pi_t) < 0$

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t) - \gamma\pi_t + \varepsilon_{t+1} \quad (13)$$

$$\pi_t = \delta + \theta h_{t+1} \quad (14)$$

$$\varepsilon_{t+1} / \Omega_t \sim N(0, h_{t+1}^2) \quad (15)$$

$$h_{t+1}^2 = \phi_0 + \sum_{j=1}^J \phi_j \varepsilon_{t+1-j}^2 + \lambda z_t \quad (16)$$

where Ω_t represents the conditional information available at time t and z_t is some vector of weakly exogenous and possibly lagged dependent variables.¹⁸ This specification provides a very simple model in which the conditional variance of the regression in an autoregressive form enters as a regressor and the forward risk premium, a linear function of the standard deviation of the conditional variance, affects the future change of the spot exchange rate. The empirical results based on this approach, however, do not support UIP. The β estimate still remains significantly smaller than unity and is often negative. Also, there is little evidence for the ARCH-type regression innovations being strongly related to the forward risk premium. These unsatisfactory results imply that the ARCH-type approaches are not appropriate for capturing the forward risk premium.

Another approach, called a forward risk premium model with structural heteroscedasticity, links the conditional variance of the disturbance term to the regressors. Koning and Straetmans (1997) modeled the observed persistence in volatility by linking the conditional variance to the squared forward premium:

$$h_{t+1}^2 = (f_t - s_t)^2 \sigma_\eta^2 + \sigma_u^2 \quad (17)$$

¹⁸ Domowitz and Hakkio (1985) used $(S_{t+1} - S_t)/S_t$ and $(F_t - S_t)/S_t$ instead of $\ln S_{t+1} - \ln S_t$ and $\ln F_t - \ln S_t$ in equation (13).

where σ_η^2 , σ_u^2 are the variances of white noise process η_t and u_t , respectively. In this case, the forward risk premium is determined by the forward premium. They estimate the following equation using the squared forward premium as a forward risk premium proxy.

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t) - \gamma\pi_t + \varepsilon_{t+1} \quad (18)$$

$$\pi_t = (f_t - s_t)^2 \quad (19)$$

The results of this approach have been mixed and have not been found to be robust when applied to different data sets and sample periods.

As noted by Lewis (1994), for credible degrees of risk aversion, the forward risk premium approach has been so far unable to explain any significant degree of the variation in the future change of the spot exchange rate. In other words, the authors have not yet succeeded in giving a general explanation for the UIP puzzle through the forward risk premium approach.

2) The irrational forecasting behavior approach¹⁹

The second approach to explaining the UIP puzzle relies on the idea that there may exist a systematic bias among market participants for predicting the expected change in the spot exchange rate. Frankel and Froot (1990) provided theoretical and empirical evidence for this conjecture. Following their study, suppose that the expectational errors ($E_t s_{t+1} - s_{t+1}$) in

¹⁹ In practice, it is not easy to differentiate between a forward risk premium and a systematic forecasting error from observed data. Marton (1994) suggests a way to determine the relative importance of a forward risk premium and a systematic expectational error in explaining the UIP puzzle. The study takes joint tests of the three parity conditions-UIP, PPP (purchasing power parity), and RIP (real interest parity)-by relating nominal and real interest differentials and inflation differentials to the same set of variables currently known to investors.

the spot exchange rate are systematically correlated with the forward premium.²⁰ This relationship can be expressed in the following equation:

$$E_t s_{t+1} - s_{t+1} = \theta + \gamma(f_t - s_t) + \mu_{t+1} \quad (20)$$

where μ_{t+1} is a white noise process. If we abandon the rational expectations assumption, equation (7) becomes:

$$E_t s_{t+1} - s_t = \alpha + \beta(f_t - s_t) + \varepsilon_{t+1} \quad (21)$$

where $E_t s_{t+1}$ is not a rational expectation but an irrational expectation, which allows for systematic forecast errors, and ε_{t+1} is a white noise process. Combining the above two equations yields:

$$\Delta s_{t+1} = (\alpha - \theta) + (\beta - \gamma)(f_t - s_t) + u_{t+1} \quad (22)$$

In the above equation, even if β is unity as provided by UIP, a negative slope coefficient ($\beta - \gamma$) is possible when γ is greater than unity.

Frankel and Froot (1987), Froot and Frankel (1989) have presented some evidence suggesting that a market participants' expectations could be systematically biased. They tested equation (20) using market survey data to measure the exchange rate expectation. The results showed that estimates of γ in expression (20) are all positive and around 3.0 for most of the data sets studied.²¹ Such a value would clearly correspond to a slope estimate of -2.0 in equation (7) if the true β was unity as implied by UIP. These results imply that a deviation

²⁰ Forecast errors in exchange market should be random if UIP holds.

²¹ The estimates are, respectively, 1.4903, 4.8067, 6.0725 and 3.2452 for Economist data, MMS 1-month, MMS 3-month and AMEX survey data.

of the slope estimate from unity is mainly due to the correlation between the forward premium and the expectational errors in the spot exchange rate rather than the high volatility of the forward risk premium. They attributed this finding to the irrational behavior arising from the presence of heterogeneous traders in the market.

However, this approach is known to have a major problem. It is not easy to rationalize why people continue to forecast the future spot rate irrationally in a systematic way. In that regard, McCallum (1994) insisted that the adaptive expectation suggested by Frankel and Froot (1987) could compare favorably with this approach.

3) The “peso problem” approach²²

Even when expectations are rational, the forward premium (or the interest rate differential) may be systematically correlated with the expectational errors in the spot exchange rate over a short horizon through the “peso problem” effect. A “peso problem” arises when market participants anticipate the possibility of a large change in economic fundamentals that does not occur in the sample period.²³ This phenomenon will tend to produce a skewness in the distribution of the forward exchange rate forecast errors even if agents’ expectations are rational, and thus may generate the apparent evidence of systematic forecast errors. Following Lewis (1994), the expected future exchange rate (E_{t+1}) is based

²² McCallum (1994) also considers how rational learning about a possible past shift in the economic regime can cause rational systematic forecast errors. This idea is almost the same as the peso problem except that the former concerns past events and the latter concerns future events.

²³ Milton Friedman used this term to explain significantly higher Mexican peso interest rates during the early 1970s than U.S. dollar interest rates even though the exchange rate had been fixed for a decade. He argued that higher Mexican peso interest rates reflected an expected devaluation of the peso through covered interest parity. The Mexican peso was devaluated in the 1970s (Lewis (1994)).

upon the current regime (C), and an alternative regime (A) that could be realized in the future according to:

$$E_t s_{t+1} = (1 - p_t) E_t(s_{t+1} | C) + p_t E_t(s_{t+1} | A) \quad (23)$$

where p_t is the probability that the current regime (C) will shift to an alternative regime (A). For expositional simplicity, suppose that regime A corresponds to a devaluation in home currency, so that $E_t(s_{t+1} | A) > E_t(s_{t+1} | C)$, but the regime shift does not occur so that in period $t+1$ the exchange rate is in fact generated by the current regime (C). Therefore, the expectational errors (ρ_{t+1}) will be:

$$\rho_{t+1} = s_{t+1}^C - E_t s_{t+1} = (s_{t+1}^C - E_t(s_{t+1} | C)) + p_t (E_t(s_{t+1} | C) - E_t(s_{t+1} | A)) \quad (24)$$

where s_{t+1}^C indicates a realization of the exchange rate from regime C.

Now, under the “peso problem” effect, the UIP condition can be rewritten as follows:

$$s_{t+1} - s_t = f_t - s_t + \rho_{t+1} \quad (25)$$

We can test equation (25) using the following regression equation:

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \theta \rho_{t+1} + u_{t+1} \quad (26)$$

where u_{t+1} is a white noise process. The OLS estimator of β is:²⁴

$$^{24} \hat{\beta} = \frac{\text{cov}(f_t - s_t, s_{t+1} - s_t)}{\text{var}(f_t - s_t)} = \frac{\text{cov}(f_t - s_t, f_t - s_t + \rho_{t+1})}{\text{var}(f_t - s_t)} = 1 + \frac{\text{cov}(f_t - s_t, \rho_{t+1})}{\text{var}(f_t - s_t)} = 1 + \frac{\text{cov}(E_t s_{t+1} - s_t, \rho_{t+1})}{\text{var}(E_t s_{t+1} - s_t)}$$

$$\hat{\beta} = \frac{\text{cov}(f_t - s_t, s_{t+1} - s_t)}{\text{var}(f_t - s_t)} = 1 - \beta_{pe}, \quad \text{where} \quad \beta_{pe} = - \left(\frac{\text{cov}(E_t s_{t+1} - s_t, \rho_{t+1})}{\text{var}(E_t s_{t+1} - s_t)} \right) \quad (27)$$

If the estimate of β is to be less than unity, the covariance between the expected change in spot exchange rates and the expectational errors must be negative. To show this possibility, assume that the conditional forecasts upon each regime are uncorrelated. Then the numerator of equation (27) can be displayed as follows:²⁵

$$\text{Cov}(E_t \Delta s_{t+1}, \rho_{t+1}) = p_t [(1 - p_t) \text{Var}(E_t \Delta s_{t+1}^C) - p_t \text{Var}(E_t \Delta s_{t+1}^A)] \quad (28)$$

The covariance between the expected change in spot exchange rates and the expectational errors can be negative when the probability-weighted variance of the exchange rate in regime A exceeds its counterpart in regime C. If the probability (p_t) of regime A is sufficiently large, the covariance will be negative.

The intuition behind this result is straightforward. During an anticipated future change from regime C to regime A, the market expects a weaker domestic currency than is realized *ex post*. The forward premium reflects the expected change in the exchange rate that, in turn, depends upon the probability (p_t) of regime A. However, since the realized regime is in fact C, the expectational errors tend to reflect unexpected systematic appreciation in the domestic currency. This interaction generates a negative covariance between the expectational errors and the forward premium when the probability of regime A in the market

²⁵ $\text{cov}(E_t s_{t+1} - s_t, \rho_{t+1}) = \text{cov}((1 - p_t) E_t \Delta s_{t+1}^C + p_t E_t \Delta s_{t+1}^A, s_{t+1}^C - E_t s_{t+1}^C + p_t (E_t s_{t+1}^C - E_t s_{t+1}^A)) =$
 $= \text{cov}((1 - p_t) E_t \Delta s_{t+1}^C + p_t E_t \Delta s_{t+1}^A, p_t (E_t \Delta s_{t+1}^C - E_t \Delta s_{t+1}^A)) = p_t ((1 - p_t) \text{var}(E_t \Delta s_{t+1}^C) - p_t \text{var}(E_t \Delta s_{t+1}^A))$

is sufficiently high. As the market believes regime A less likely, the negative covariance between the expectational errors and the forward premium disappears.

Evans and Lewis (1995) studied this possibility by estimating a switching regime model that allows for potential “peso problem” effect in three major U.S. dollar exchange rates. They showed that the Fama (1984) coefficient is biased downward as a result of the “peso problem” effect. Additionally, they suggested that the standard inference techniques based upon assuming that the expectational error and the forward premium are uncorrelated could be misleading when applied to equation (7). This suggestion implies that a “peso problem” effect may introduce an important component of the deviation from unity in the estimated slope coefficient. However, they also showed that peso problems alone could not explain all the behavior of systematic expectational errors. Even after adjusting for the peso problem bias in coefficients and variances, the remaining component of predictable returns remains sizable.

4) The policy intervention approach

The basic idea of this approach is that policy makers in both home and foreign countries have some tendency to smooth the movements of exchange rates. McCallum (1994) suggested that even though UIP holds, the estimate of β can be less than unity as long as policy-makers manage the interest rate differential to systematically smooth the interest rate and exchange rate changes.²⁶ For example, when exchange rates are expected to fall,

²⁶ More recently, Anker (1999) reached a similar conclusion in the context of a monetary model for a small open economy in which the central bank reacts to exogenous forward risk premium shocks.

policy makers will try to smooth the change in spot exchange rate by taking expansionary monetary policy. These policy responses might be represented by the following policy response equation:

$$i_t - i_t^* = \lambda(s_t - s_{t-1}) + \gamma(i_{t-1} - i_{t-1}^*) + \zeta_t \quad (29)$$

where λ , γ indicate the intensity of the policy response to a change in the spot exchange rate and the interest differential, respectively, and ζ_t denotes an interest rate shock. Recall that under the rational expectations assumption with a forward risk premium shock, UIP is written as follows:

$$E_t s_{t+1} - s_t = i_t - i_t^* - \pi_t \quad (30)$$

where π_t denotes the forward risk premium shock, which we will assume follows a white noise process. Combining equations (29) and (30) yields:

$$E_t \Delta s_{t+1} = \lambda \Delta s_t + \gamma(i_{t-1} - i_{t-1}^*) + \zeta_t - \pi_t \quad (31)$$

The undetermined coefficients method gives the solution for Δs_t :

$$\Delta s_t = -\frac{\gamma}{\lambda}(i_{t-1} - i_{t-1}^*) - \frac{1}{\lambda}\zeta_t + \frac{1}{\lambda + \gamma}\pi_t \quad (32)$$

If γ and λ are both positive, the coefficient of $(i_{t-1} - i_{t-1}^*)$ will be negative. Hence, for example, if $\gamma=0.4$ and $\lambda=0.2$, we get a coefficient of -2 in the above relation, which is similar to the estimates of β typically reported for the UIP regression.

Thus, the policy intervention approach in which monetary authorities manage interest rate differentials so as to resist rapid changes in exchange rates and interest differentials is attractive conceptually and capable of partially explaining the UIP puzzle. However, Michael (2000) estimated the modified UIP regression developed by McCallum using the extended data of McCallum's and found that the estimated coefficient is not consistent with the UIP condition. Additionally, the policy reaction function does not include some important variables such as inflation and output.

To generalize McCallum's idea, Meredith and Chinn (1998) incorporated output and inflation into the policy response function and tested the UIP condition using the interest differential with considerably longer maturity data than those employed in the previous studies.^{27,28} For financial instruments with maturities ranging from 5 to 10 years, all of the coefficients on the interest differentials in the UIP regressions have the correct sign. They argue that for relatively short horizons, the failure of UIP may result from temporary forward risk premium shocks in the presence of an endogenous monetary policy that is characterized by "leaning against the wind" of prevailing exchange rate movements. In the long run, exchange rates are driven by macroeconomic "fundamentals" leading to a relation between interest rate differentials and exchange rates that should be more consistent with UIP. However, almost all of the coefficients on the interest differentials in their paper are still much below unity.

²⁷ Berk and Klass (2001) also use long-term interest rates data to test the UIP relationship. Additionally, they employ exchange rate expectations derived from PPP instead of actual exchange rates. The results do not support the validity of UIP among major industrialized countries' currencies.

²⁸ A notable aspect of almost all published studies is that UIP has been tested using financial instruments with relatively short maturities, generally of 12 months or less.

Kirikos (2000) combined the policy makers' market intervention approach with the "peso problem" approach. In particular, he showed that when policy-makers postulate a policy rule, which allows for discrete regime shifts in managing the interest rate differential, then the dynamic behavior of the exchange rate and the interest rate differential could be described by a stochastic segmented trends representation. In this setting, he tested the UIP hypothesis in terms of a statistical test of the cross-equation restrictions on the parameters of a Markov switching regimes representation and suggested that the UIP condition cannot be rejected for the currencies of Greece, Italy, and Portugal relative to the U.S. dollar.²⁹

In conclusion, none of the existing approaches to explaining the UIP puzzle completely resolves the puzzle. However, the empirical results using the "peso problem" approach and the policy intervention approach seem most consistent with UIP. They yield positive coefficients that are close to unity. McCallum's idea of combining a policy response function with UIP may be the most promising for guiding further research. McCallum assumed that policy-makers respond smoothly to the change of the spot exchange rate. Hence, the monetary policy response function is a linear function of the change of the spot exchange rate at the current period and one period lagged interest rate differentials. However, suppose that the monetary authority's response to the change in spot exchange rates is different for a small deviation from the long-term equilibrium than it is for a large deviation. This idea suggests that the deviation from the long-run equilibrium follows a nonstationary process within a certain band but shows mean-reverting behavior outside the band.

The existing approaches to explaining the UIP puzzle are not appropriate to cope with this specific characteristic. Instead, the method to estimate the long run equilibrium

²⁹ See Engel and Hamilton (1990), Kaminsky (1993), Evans and Lewis (1995), and Kirikos (2000).

relationship between the change in the exchange rate and the interest differential must consider the level-dependence of the stochastic process. As we will explain in the next section, a threshold cointegration model can capture this kind of discrete adjustment of policy interventions. The policymakers do not respond to small deviations from the long run relationship between the change in the exchange rate and the interest differential. However, once the deviation from the long-run equilibrium exceeds the band, the mean reverting adjustment quickly occurs.

CHAPTER 4. UIP AND THRESHOLD COINTEGRATION

In the previous section, we presented several rationales for the UIP puzzle. From now on, we take a completely different approach. The main idea is that the UIP puzzle is due to a failure to adequately model the stochastic features of the data generating process. To model the particular time series behavior of spot exchange rates and forward exchange rates, the threshold cointegration model is introduced and estimated.

1. Long-run equilibrium relationship and threshold cointegration

In general, linear combinations of nonstationary time series will also be nonstationary. However, a set of nonstationary economic variables may be linked by long-run linear equilibrium relationships and, thus, have a tendency not to wander too far from each other in the long-run. More formally, there may exist linear combinations of the variables that are stationary. If this is true, these variables are called “cointegrated” variables, a concept introduced by Granger (1981) and subsequently developed by Engle and Granger (1987). Typically, the cointegration error process, i.e., the deviation from the long-run equilibrium, is assumed to form a linear process, e.g., an AR (p) process. In this case, adjustments to disequilibrium are linear. The main features of a linear adjustment process are that the adjustment process is symmetric around the mean and the speed of adjustment is constant. The cointegrated variables are also represented in terms of an error correction model. In this case, the cointegration relationship among variables gives us a “long-run” equilibrium

relationship. In contrast, the error correction representation provides short-run dynamics of these variables that show the adjustment process to a deviation to the long-run equilibrium.

However, when there are market frictions or discrete policy responses of policy makers, a deviation can persist for a very long time without evidence of mean reverting behavior. For example, transportation costs for a commodity traded in spatially separate markets or transaction costs related to the purchase or sale of the commodity can allow apparent arbitrage opportunities to persist if the price differences are too small to compensate for these costs. Consequently, the behavior of the commodity price may follow a random walk within a certain band, but exhibit mean reversion outside the band. Or, there may be cases where policymakers' policy responses are asymmetric or discrete so that adjustment to a deviation from a long-run relationship may be nonlinear. Balke and Fomby (1997) present exchange rate management via target zones, commodity price stabilization programs, and the Federal Reserve's Fed Funds rate and Discount rate policies as examples of possible asymmetric and discrete policy interventions. For example, when exchange rates are within a target zone, central banks may choose not to intervene in foreign exchange markets and, thus, exchange rates will seem to be nonstationary during certain intervals. However, once exchange rates break the target zone, central banks may intervene in the foreign exchange markets and, thus, the exchange rates will show mean reverting behavior during certain intervals.

Balke and Fomby (1997) proposed to model this kind of nonlinear adjustment to a deviation from the long-run equilibrium in terms of a threshold cointegration method.³⁰ This method recognizes that no adjustment towards long-run equilibrium takes place until the

³⁰ See Balke and Fomby (1997) for more detailed explanation of threshold cointegration.

deviation reaches a certain threshold, at which point a mean reverting adjustment process begins to work. The essential distinction between the threshold cointegration process and the conventional cointegration process is that in the case of threshold cointegration the cointegration error process is nonlinear, following a threshold autoregressive process rather than a linear autoregressive process. The definition of a threshold autoregressive (i.e., TAR) model will be made more precise in the modeling part of this paper. For now, we simply note that in a linear AR (p) model of a stationary process Y_t , the partial derivatives dY_t/dY_{t-j} , $j = 1, \dots, p$ are constants; in a TAR (p) model of a stationary process Y_t , at least some of these partial derivatives are state-dependent.

The threshold cointegration model has been applied to study various topics such as purchasing power parity (PPP), the long-term Fisher effect, the Fed Funds rate and the Discount rate, the term structure of interest rates, covered interest parity (CIP), and uncovered interest parity (UIP). In the next section, we will provide a review of this empirical literature.

2. Literature review on the application of threshold cointegration

Balke and Fomby's (1997) introduction of the threshold cointegration model included an application to the relationship between the Fed Funds rate and the Discount rate. Assume that these rates are unit root processes and the Federal Reserve conducts monetary policy to maintain a particular linear combination of these rates (i.e., the spread) close to zero. Consequently, the two rates are cointegrated. However, suppose that the Federal Reserve only takes action when the spread is sufficiently far from zero. Then the spread will

follow a threshold autoregression rather than a linear autoregression and, by definition, the Fed Funds rate and Discount rate will follow a threshold cointegration process. Using monthly data from January 1955 to December 1990, Balke and Fomby (1997) found that the Fed Funds rate and the Discount rate are cointegrated and can be characterized by threshold cointegration. As long as the spread between the two rates is in the range $(-0.2, 1.6)$, there does not appear to be any mean reversion. However, when the spread is greater than 1.6 percentage points or less than -0.2 percentage points there is evidence of mean reversion (to an equilibrium band rather than to an equilibrium point).

Michael, Nobay and Peel (1997) investigated the possibility that incorporating transaction costs into equilibrium models of real exchange rate determination may cause nonlinear adjustment toward purchasing power parity (PPP). They argue that transaction costs may prevent agents from adjusting continuously, which leads to discrete adjustment that can be approximated by a threshold model. Hence, the equilibrium error is modeled as a threshold autoregression that is mean-reverting outside a given range and has a unit root inside this range. They characterized this nonlinear adjustment process in terms of a smooth transition autoregressive (STAR) model because of the aggregation property of heterogeneous agents. The data set used in their study includes monthly data during the interwar period and annual data during two centuries for the United Kingdom, United States, France, and Germany. The results of their study clearly reject the linear framework in favor of an exponential smooth transition autoregressive process. The systematic pattern in the estimates of the nonlinear models provides strong evidence of mean reverting behavior of PPP deviations and helps explain the mixed results of previous studies.

Weidmann (1997) used German data to study the possibility that the bivariate stochastic process governing the joint behavior of inflation and interest rates depends on the level of the variables and that the long-run Fisher effect should be modeled using threshold cointegration. Whenever the inflation rate falls in a certain band of tolerable inflation, the policy maker does not conduct an active policy. However, if the inflation rate falls outside the band, the policy maker actively pursues a monetary policy that aims to bring inflation back inside the band. Consequently, inflation and interest rates will move like independent random walks within the band but they will show mean reversion if inflation breaches the threshold. Using a self-exciting threshold autoregressive (SETAR) model, he showed that when inflation is between 1.5 and 4.7 percent, the Bundesbank does not take a deliberate action, but it engages in a restrictive policy to bring inflation back within the band if inflation exceeds the band.

Balke and Wohar (1998) examined the dynamics of deviations from covered interest parity (CIP) using daily data on the U.K. spot and forward exchange rates against the U.S. dollar and Euro-deposit interest rates over the period, January 1974 to September 1993. In order to examine the persistence of CIP deviations, they used the threshold cointegration method in which deviations from covered interest parity that are outside the transaction costs band shows less persistence than those that lie inside the band.

Coakley and Fuertes (1999) used threshold cointegration to explore the stationarity of excess foreign exchange returns and the rehabilitation of UIP for the currencies of the G10 countries and the Swiss franc during a floating exchange rate period, 1976 to 1997. They insisted that transaction costs hinder agents from adjusting to a small deviation from the long-run equilibrium. Hence, the excess returns show unit root behavior within a certain band

and mean reverting behavior outside the band. Their results showed that threshold autoregressive dynamics are able to capture a relatively high degree of persistence of a deviation within a certain band and quick mean reverting behavior outside the band. However, the estimates of the threshold suggest a no-arbitrage band of some 2.4 percent on average, which readily exceeds the bid-ask spread. They rationalize this difference by noise (or chartist) trading behavior.

In the next section, we will provide the details of our threshold cointegration approach to the UIP puzzle. Our approach differs from Coakley and Fuertes (1999) in several ways. First, Coakley and Fuertes (1999) studied not UIP but the forward rate unbiased hypothesis (FRUH). However, our study is designed to explain the UIP puzzle directly. Second, Coakley and Fuertes (1999) apply the univariate procedure suggested by Hansen (1997). In contrast, we will follow the multivariate procedure provided by Tsay (1998) and Lo and Zivot (2000) for modeling the threshold cointegration relationship. Hence, we will provide a threshold vector error correction model (TVECM) based on the Granger's Representation Theorem (Engle and Granger (1987)). The error correction model provides a way to simultaneously capture the long-run equilibrium relationship and the disequilibrium adjustment process toward the long-run equilibrium. Third, we will apply a model specification test suggested by Lo and Zivot (2000) to see if the band TVECM is appropriate for modeling threshold cointegration relationship for UIP. Coakley and Fuertes (1999) study was rather *ad hoc* and did not apply a model specification test to find the appropriate number of threshold regimes.

3. Methodology

Without transaction costs, the UIP condition is tested using the following regression equation:

$$s_{t+1} - s_t = \alpha + \beta(i_t - i_t^*) + \varepsilon_{t+1} \quad (33)$$

where s_t is the log of the nominal exchange rate, i_t and i_t^* are the domestic and foreign interest rate, respectively, and ε_{t+1} is a white noise process. In empirical works, CIP is assumed to hold so that equation (33) is often replaced by the following regression:

$$s_{t+1} - s_t = \alpha + \beta(f_t - s_t) + u_{t+1} \quad (34)$$

Generally, the joint null hypothesis to be tested is $H_0: \alpha=0, \beta=1$, and u_{t+1} has a conditional mean of zero. Most empirical studies have shown that the above joint null hypothesis is almost always rejected and, especially, the estimate of β is significantly negative. The rejection of the hypothesis of UIP specified in terms of the regression equation (34) implies either that the data do not support the UIP condition or that equation (34) is misspecified for modeling the UIP condition. In this paper, we will focus on the latter conjecture. In order to capture the behavior of the spot and forward exchange rates more accurately, the general bivariate vector autoregressive (VAR) model is considered. If we define $Y_t = (s_t, f_t)'$, the bivariate VAR model for Y_t can be expressed in the form:

$$Y_t = A_0 + \sum_{i=1}^p A_i Y_{t-i} + \varepsilon_t \quad (35)$$

where ε_t is a bivariate vector white noise process, p is the autoregressive order, A_0 is a 2×1 column vector, and the A_i 's are 2×2 parameter matrices.

Equation (35) can be reparameterized as:

$$\Delta Y_t = A_0 - \Pi Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t \quad (36)$$

where $\Pi = (I_2 - A_1 - A_2 - \dots - A_p)$ and $\Gamma_i = -A_{i+1} - A_{i+2} - \dots - A_p$ for $i=1,2,\dots,p-1$. If the components of Y_t are $I(1)$ and cointegrated with a cointegrating vector $\delta=(1,-1)$, then the vector error correction model for UIP can be expressed as follows:

$$\Delta Y_t = A_0 - \beta \delta Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t \quad (37)$$

where

$$\beta \delta' = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} (1, -1) = \Pi \quad (38)$$

Rewriting equation (37) in algebraic form, it becomes:

$$\begin{pmatrix} \Delta s_t \\ \Delta f_t \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} (f_{t-1} - s_{t-1}) + \begin{pmatrix} \sum_{i=1}^{p-1} \theta_{s_i} \Delta s_{t-i} + \sum_{i=1}^{p-1} \theta_{f_i} \Delta f_{t-i} \\ \sum_{i=1}^{p-1} \phi_{s_i} \Delta s_{t-i} + \sum_{i=1}^{p-1} \phi_{f_i} \Delta f_{t-i} \end{pmatrix} + \begin{pmatrix} \varepsilon_{s_t} \\ \varepsilon_{f_t} \end{pmatrix} \quad (39)$$

When $p=1$, the first row of equation (39) is the same as equation (34). Hence, equation (34) can be interpreted as a model that ignores the effect of the lagged differences in the spot and forward exchange rates on the current change in the spot exchange rate. Barnhart and

Szakmary (1991) estimated equation (39) for four developed countries (Canada, Germany, United Kingdom, and Japan) and found that β_i is still significantly negative when $p=2$. When $p=1$, Zivot (1997) found similar results, though he did not take into account the United Kingdom in his work.

Now, let us introduce the threshold cointegration approach to the UIP puzzle. Threshold cointegration behavior between the spot and forward exchange rates can be investigated in terms of a threshold vector error correction model (TVECM) or a threshold autoregressive (TAR) model.³¹ The TVECM or TAR model allow the adjustment process of a deviation to the long-run equilibrium path to depend on the current state of the system. The TVECM or TAR model we will be interested in will have the property that within a certain band between an upper threshold and a lower threshold, there is no tendency to move toward the equilibrium path, so that deviations from UIP may exhibit unit root behavior within this band. Outside the band, the process will be mean reverting.³² If the mean reverting behavior is toward an equilibrium line we call the TVECM (TAR model) an “equilibrium TVECM (TAR model)”. If the mean reverting behavior is simply toward the band, we call the TVECM (TAR model) a “band-TVECM (TAR model)”. In empirical studies on the threshold cointegration relationship between economic variables, the TAR model has been widely used relative to the TVECM. However, if there are restrictions imposed by the

³¹ See Tong (1983, 1990), Tsay (1989), Chan (1990), Chan and Tong (1990), and Hansen (1996, 1997) for more details.

³² Threshold cointegration can also be characterized in terms of a smooth transition autoregressive (STAR) model. Here, adjustment takes place in every period, but the speed of adjustment varies with the extent of the deviation from parity. In contrast with the TAR model, regime changes occur gradually rather than abruptly. See Chan and Tong (1986), Saikkonen and Luukkonen (1988), Granger and Terasvirta (1993), and Terasvirta, et. al. (1994) for more details.

multivariate structure, multivariate procedures are more appropriate than univariate procedures.

It will be useful at this moment to introduce a basic bivariate three regime threshold vector error correction model (TVECM) for the threshold cointegration behavior of UIP.³³ The bivariate threshold vector autoregressive (TVAR) model for Y_t can be expressed in the form:

$$Y_t = (A_0^1 + \sum_{i=1}^p A_i^1 Y_{t-i} + \varepsilon_t^1) I_{1t}(z_{t-d} \leq \gamma_1) + (A_0^2 + \sum_{i=1}^p A_i^2 Y_{t-i} + \varepsilon_t^2) I_{2t}(\gamma_1 < z_{t-d} \leq \gamma_2) + (A_0^3 + \sum_{i=1}^p A_i^3 Y_{t-i} + \varepsilon_t^3) I_{3t}(z_{t-d} > \gamma_2) \quad (40)$$

where ε_t is a bivariate vector white noise process, p is the autoregressive order, the A_0^j 's are 2×1 column vectors, and the A_i^j 's are 2×2 parameter matrices that apply in regime j , $j = 1, 2, 3$ and lag $i = 1, 2, \dots, p$.^{34,35} The threshold variable z_t is assumed to be known whereas the delay parameter d , the autoregressive order p and the threshold values, γ_1 and γ_2 , are potentially unknown. The three-regimes are determined by the "delay parameter" d , which is a positive integer, the parameters $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ and the indicator functions $I_{1t}(\gamma, d)$, $I_{2t}(\gamma, d)$, $I_{3t}(\gamma, d)$ according to:

$$-\infty = \gamma_0 < \gamma_1 < \gamma_2 < \gamma_3 = \infty, \quad \text{and}$$

$$I_{jt}(\gamma_{j-1} < z_{t-d} \leq \gamma_j) = \begin{cases} 1 & \text{if } \gamma_{j-1} < z_{t-d} \leq \gamma_j, \quad j = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases} \quad (41)$$

³³ The following threshold cointegration procedure for bivariate cases is mainly based on Lo and Zivot (2000).

³⁴ ε_t can be a martingale difference sequence, which allows for heteroscedasticity.

³⁵ Tong (1983) and Tsay (1989) suggest that the AR order for each regime needs not to be same. However, Hansen (1999) explains that testing for linearity and the number of regime in SETAR(m) models becomes more complex if a different AR order for each regime is allowed.

Equation (40) can be reparametrized as:

$$\begin{aligned} \Delta Y_t = & (A_0^1 - \Pi^1 Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i^1 \Delta Y_{t-i} + \varepsilon_t^1) I_{1t}(z_{t-d} \leq \gamma_1) + (A_0^2 - \Pi^2 Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i^2 \Delta Y_{t-i} + \varepsilon_t^2) \\ & I_{2t}(\gamma_1 < z_{t-d} \leq \gamma_2) + (A_0^3 - \Pi^3 Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i^3 \Delta Y_{t-i} + \varepsilon_t^3) I_{3t}(z_{t-d} > \gamma_2) \end{aligned} \quad (42)$$

where $\Pi^j = (I_2 - A_1^j - A_2^j - \dots - A_p^j)$ and $\Gamma_i^j = -A_{i+1}^j - A_{i+2}^j - \dots - A_p^j$ for $j=1,2,3$ and $i=1,2,\dots,p-1$. If the components of Y_t are $I(1)$ and cointegrated with common cointegrating vector $\beta'=(1,-1)$ within each regime, and if the errors in each regime have a common covariance matrix, then UIP is expressed as follows:

$$\begin{aligned} \Delta Y_t = & (A_0^1 - c^1 \beta' Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i^1 \Delta Y_{t-i}) I_{1t}(z_{t-d} \leq \gamma_1) + (A_0^2 - c^2 \beta' Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i^2 \Delta Y_{t-i}) \\ & I_{2t}(\gamma_1 < z_{t-d} \leq \gamma_2) + (A_0^3 - c^3 \beta' Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i^3 \Delta Y_{t-i}) I_{3t}(z_{t-d} > \gamma_3) + \varepsilon_t \end{aligned} \quad (43)$$

where

$$c^j \beta' = \begin{pmatrix} c_1^j \\ c_2^j \end{pmatrix} (1, -1) = \Pi^j, \quad j = 1, 2, 3 \quad (44)$$

The TVECM specifies that the adjustment toward the long-run equilibrium relationship $\beta' Y_{t-1}$ is regime specific. For convenience of explanation, assume a TVAR model with $p=1$ and $d=1$, which implies a TVECM with $p=0$. Then, equation (43) can be expressed as:

$$\begin{aligned} \Delta Y_t = & (A_0^1 - c^1 \beta' Y_{t-1}) I_{1t}(z_{t-1} \leq \gamma_1) + (A_0^2 - c^2 \beta' Y_{t-1}) I_{2t}(\gamma_1 < z_{t-1} \leq \gamma_2) + \\ & (A_0^3 - c^3 \beta' Y_{t-1}) I_{3t}(z_{t-1} > \gamma_2) + \varepsilon_t \end{aligned} \quad (45)$$

In the above TVECM, the stability condition $|1 - c_1^j + c_2^j| < 1$ holds for each regime.³⁶ When

$A_0^2 = c^2 = 0$, equation (45) becomes a band TVECM:

$$\Delta Y_t = (A_0^1 - c^1 \beta' Y_{t-1}) I_{1t}(z_{t-1} \leq \gamma_1) + (A_0^3 - c^3 \beta' Y_{t-1}) I_{3t}(z_{t-1} > \gamma_2) + \varepsilon_t \quad (46)$$

Rewriting equation (46) in algebraic form, it becomes:

$$\begin{pmatrix} \Delta s_t \\ \Delta f_t \end{pmatrix} = \begin{cases} \begin{pmatrix} a_1^1 \\ a_2^1 \end{pmatrix} + \begin{pmatrix} c_1^1 \\ c_2^1 \end{pmatrix} (f_{t-1} - s_{t-1}) + \begin{pmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{pmatrix} & \text{if } z_{t-1} (= f_{t-1} - s_{t-1}) \leq \gamma_1 \\ \begin{pmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{pmatrix} & \text{if } \gamma_1 < z_{t-1} (= f_{t-1} - s_{t-1}) \leq \gamma_2 \\ \begin{pmatrix} a_1^3 \\ a_2^3 \end{pmatrix} + \begin{pmatrix} c_1^3 \\ c_2^3 \end{pmatrix} (f_{t-1} - s_{t-1}) + \begin{pmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{pmatrix} & \text{if } z_{t-1} (= f_{t-1} - s_{t-1}) > \gamma_2 \end{cases} \quad (47)$$

The stability condition for equation (47) is that the cointegration residual z_t be stationary in the outer regimes. Hence, the stability conditions require $|1 - c_1^j + c_2^j| < 1$ for $j=1,3$. The band TVECM of equation (47) has the implication that if the forward premium ($z_{t-1} = s_{t-1} - f_{t-1}$) in the exchange market are within a certain band between the lower and upper threshold, then the future spot and forward exchange rate follows a random walk without drift; if the forward premium reach the upper and lower threshold, then the future change in the spot and forward exchange rates depends on the lagged forward premium.

³⁶ This can be proved in the following way. The cointegrating residual has the regime specific AR(1) process:

$$\begin{aligned} \beta' Y_t &= \beta' A_0^j + \beta' A_1^j Y_{t-1} + \beta' \varepsilon_t = \beta' A_0^j + \beta' Y_{t-1} - (\beta' - \beta' A_1^j) Y_{t-1} + \beta' \varepsilon_t \\ &= \beta' A_0^j + \beta' Y_{t-1} - \beta' (1 - A_1^j) Y_{t-1} + \beta' \varepsilon_t = \beta' A_0^j + \beta' Y_{t-1} - \beta' c^j \beta' Y_{t-1} + \beta' \varepsilon_t \\ &= \beta' A_0^j + (1 - \beta' c^j) \beta' Y_{t-1} + \beta' \varepsilon_t \end{aligned}$$

Hence, the cointegrating residual is stable within each regime if $|1 - \beta' c^j| = |1 - c_1^j + c_2^j| < 1$

As shown up to now, finding and estimating the appropriate model are not an easy task. Balke and Fomby (1997) suggested a procedure to test the threshold cointegration for the univariate case and Lo and Zivot (2000) suggested a procedure for the multivariate case.³⁷ In this paper, we reconstruct the following test and estimation procedures to find whether the spot and forward exchange rates have the threshold nonlinearity property. First, we will test a linear cointegration relationship between the spot and forward exchange rates. Second, we will estimate a vector error correction model (VECM) and check the slope estimate of the forward premium term. Third, once the slope estimate of the forward premium term of the VECM is significantly different from unity, we will test a threshold nonlinearity relationship for the spot and forward exchange rates. Fourth, if a threshold nonlinearity relationship is found, we will estimate the appropriate band TVECM for the UIP condition. After that, we will check the parameter estimates to confirm whether the data are consistent with threshold cointegration behavior of UIP. If the data support the existence of threshold cointegration relationship, the UIP condition needs to be reinterpreted. Instead of being true globally, the UIP condition might be locally true in that it does not hold within a certain band between the upper and lower threshold but it holds outside the band. Finally, we will estimate the out-of-sample forecasts using the TVECM and other four alternative models, and compare the forecasting ability based on root-mean-square-error (RMSE) and mean-absolute-error (MAE) criteria.

³⁷ According to Lo and Zivot (2000), their suggestions can be combined as the following three-step strategy. First, test the null hypothesis of no cointegration against the alternative linear cointegration. Second, if the null of no cointegration is rejected, then test for threshold nonlinearity. Finally, if linearity is rejected, consider the model specification test of the various threshold cointegration models.

4. Empirical analysis

1) Exchange rate data

The data used in this paper consist of monthly spot (S_t) and one-month forward (F_t) exchange rates for four major currencies (the Canadian dollar, the Japanese yen, the German mark, and the British pound) obtained from Datastream.³⁸ Both rates are transformed into logarithmic form and are denoted as s_t and f_t , respectively. All currencies are denominated in terms of the U.S. dollar. The sample period covers from July 1978 through June 1996.³⁹ The spot and one-month forward exchange rates are observed on the last trading day of each month. As a result, the sampling frequency may differ from the time to maturity of one-month forward contract by one or two days. We do not expect this problem to affect the major results of this paper.⁴⁰ The reason why we focus on monthly data is that when the sampling period is shorter than the time to maturity of the forward contract, there exist serial correlation problems.

Figures 2 show the plots of the spot exchange rates for the four major currencies. All of them show the behavior of nonstationary time series data. Figures 3 are the plots of

³⁸ We would like to thank Professor Zivot for helping us get the data. The data was originally obtained from Datastream by Professor Zivot for his paper "Cointegration and forward and spot exchange rate regressions"(2000).

³⁹ The original data that we got from Professor Zivot begin in January 1976, except for forward exchange rates for the Japanese yen which begin in July 1978. We conform the starting month of the Canadian dollar, the German mark and the British pound to July 1978 based on the available data of the Japanese yen.

⁴⁰ Bekaert and Hodrick (1993) used the data in which there are no sampling errors, and found the estimated coefficient is not much different from those under the usual incorrectly sampled data.

differenced spot and forward exchange rates for each country. They exhibit the behavior of stationary time series data.⁴¹

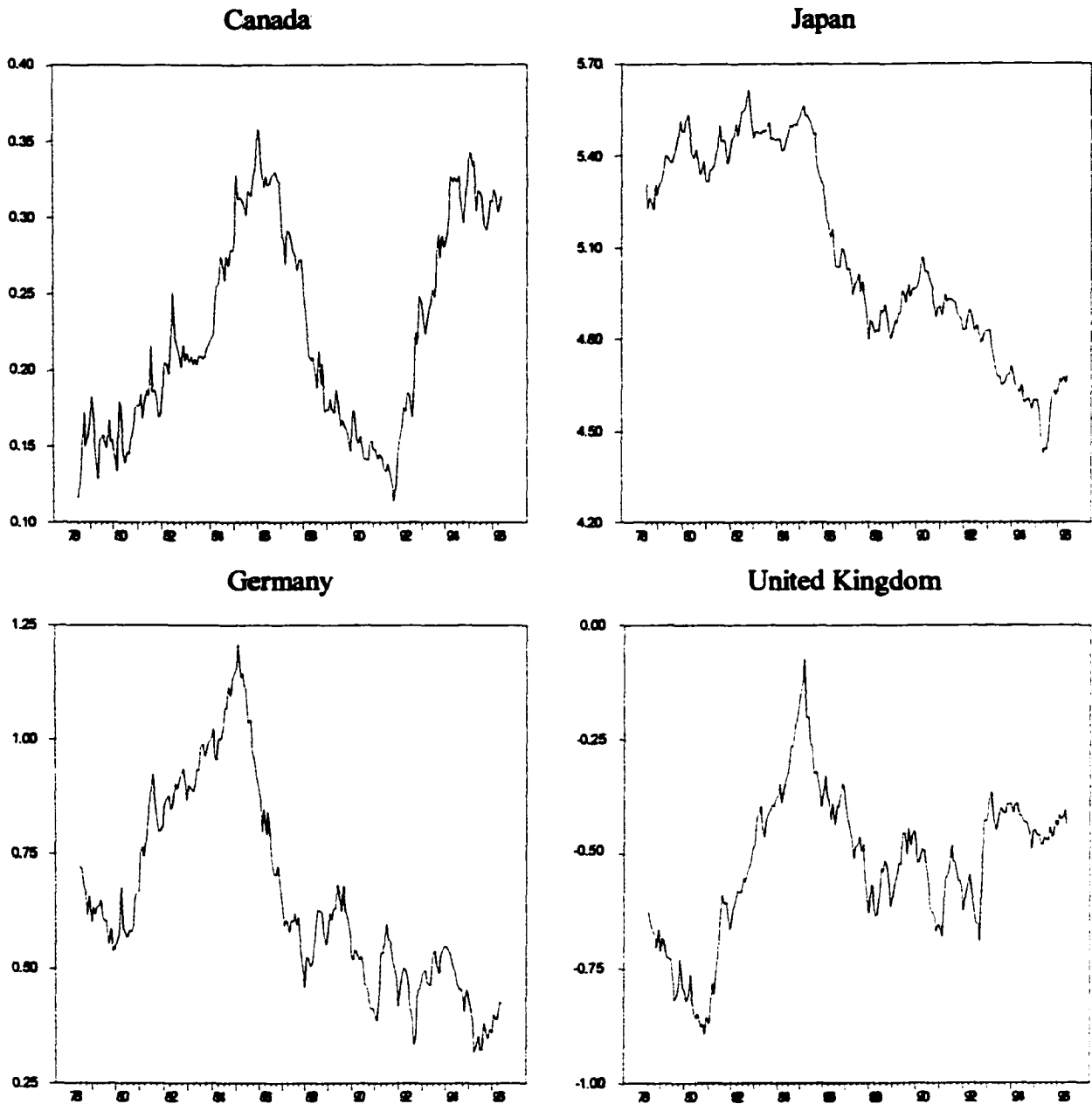


Figure 2. Monthly spot exchange rates

⁴¹ The plots of forward and differenced forward exchange rate are omitted because they are almost the same as those of spot and differenced spot exchange rate.

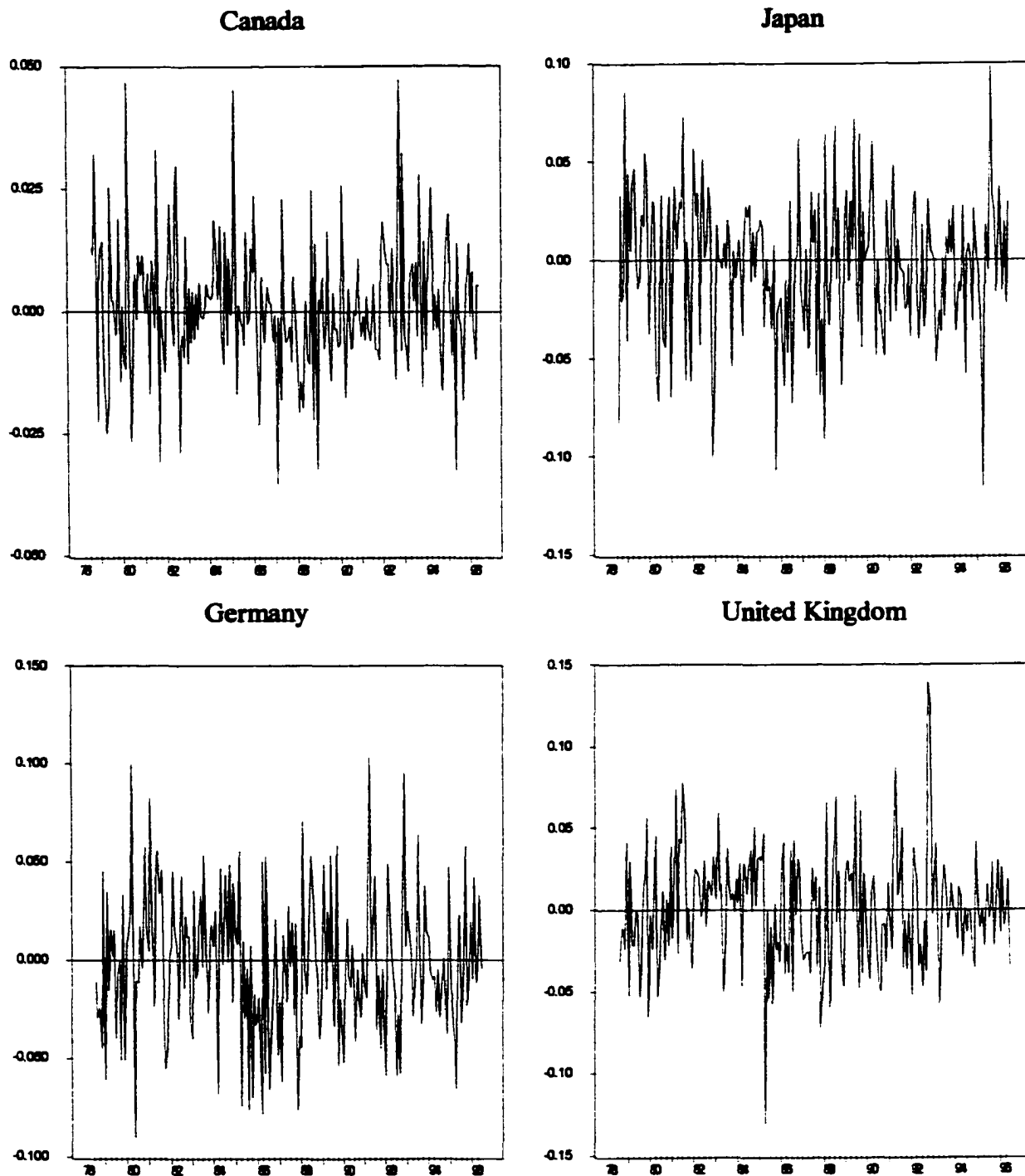


Figure 3. Differenced monthly spot exchange rates

Figures 4 and 5 show the plots of the forward premium ($f_i - s_i$) and the forecast error ($s_t - f_{t-1}$). The plots of the forecast error for each country look like stationary time series data. The plots of forward premium are, however, ambiguous. The data series are all highly autocorrelated.

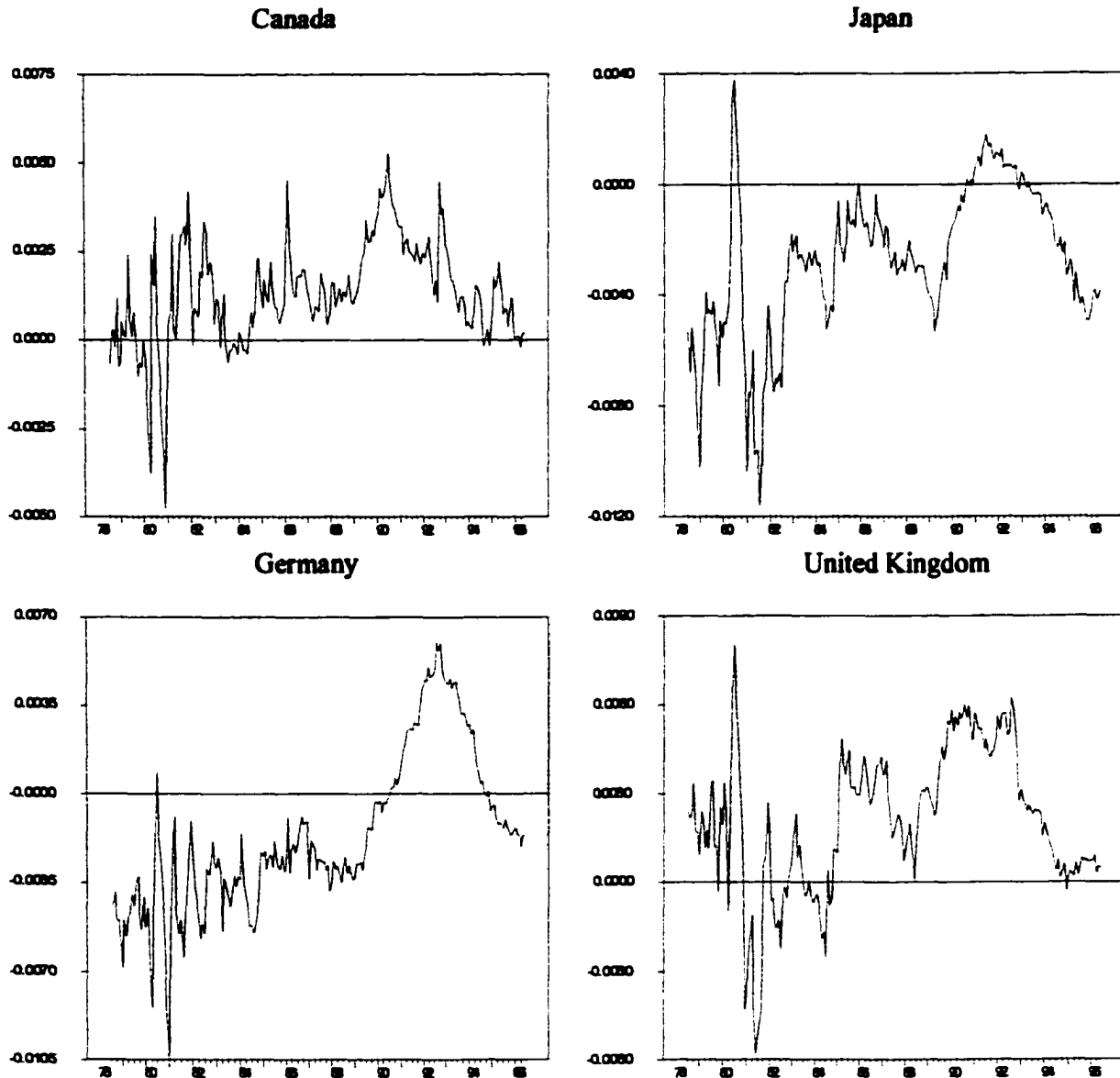


Figure 4. One-month forward premium

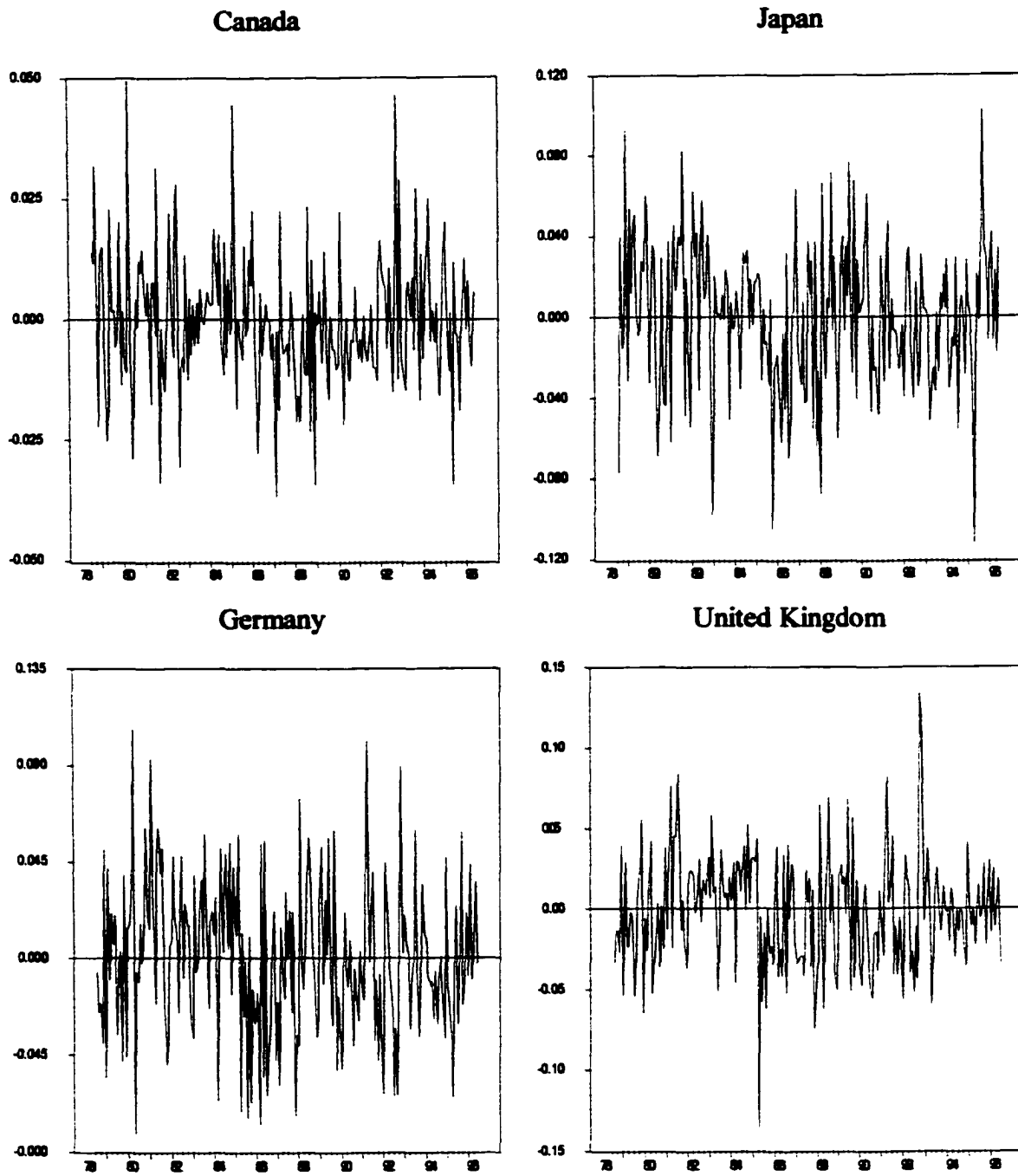


Figure 5. One-month forecast error

Table 1 provides summary statistics of the data. According to the correlation matrix of Table 1, the sign of the correlation coefficient between differenced spot rates and forward premium has the negative sign for all currencies as expected from the previous studies. The variance of the differenced spot and forward exchange rates are much bigger than that of the forward premium.

Table 1. Summary statistics for exchange rate data

Country	Series	Obs	Mean	Std. Error	Min.	Max.	Correlation Matrix			
							$s_t s_{t-1}$	$f_t f_{t-1}$	$f_{t-1} s_{t-1}$	$s_t f_{t-1}$
Canada	s_t	216	0.2253	0.0671	0.1139	0.3588				
	f_t	216	0.2266	0.0669	0.1155	0.3611				
	$s_t s_{t-1}$	215	0.0009	0.0136	-0.0352	0.0474	1.000			
	$f_t f_{t-1}$	215	0.0009	0.0137	-0.0356	0.0509	0.997	1.000		
	$f_{t-1} s_{t-1}$	216	0.0013	0.0015	-0.0048	0.0053	-0.149	-0.124	1.000	
	$s_t f_{t-1}$	215	-0.0004	0.0140	-0.0367	0.0496	0.995	0.995	-0.227	1.000
Japan	s_t	216	5.0890	0.3293	4.4257	5.6207				
	f_t	216	5.0862	0.3284	4.4225	5.6181				
	$s_t s_{t-1}$	215	-0.0029	0.0361	-0.1156	0.0984	1.000			
	$f_t f_{t-1}$	215	-0.0029	0.0360	-0.1170	0.0981	1.000	1.000		
	$f_{t-1} s_{t-1}$	216	-0.0028	0.0027	-0.0116	0.0038	-0.222	-0.217	1.000	
	$s_t f_{t-1}$	215	-0.0002	0.0367	-0.1127	0.1025	0.997	0.997	-0.286	1.000
German	s_t	216	0.6569	0.2159	0.3164	1.2109				
	f_t	216	0.6552	0.2143	0.3152	1.2080				
	$s_t s_{t-1}$	215	-0.0014	0.0356	-0.0902	0.1036	1.000			
	$f_t f_{t-1}$	215	-0.0016	0.0355	-0.0860	0.1041	1.000	1.000		
	$f_{t-1} s_{t-1}$	216	-0.0016	0.0031	-0.0104	0.0060	-0.083	-0.079	1.000	
	$s_t f_{t-1}$	215	0.0002	0.0359	-0.0818	0.1071	0.996	0.996	-0.165	1.000
United Kingdom	s_t	216	-0.5177	0.1587	-0.8934	-0.0700				
	f_t	216	-0.5157	0.1588	-0.8922	-0.0651				
	$s_t s_{t-1}$	215	0.0009	0.0348	-0.1309	0.1398	1.000			
	$f_t f_{t-1}$	215	0.0009	0.0347	-0.1317	0.1391	1.000	1.000		
	$f_{t-1} s_{t-1}$	216	0.0021	0.0024	-0.0058	0.0081	-0.236	-0.230	1.000	
	$s_t f_{t-1}$	215	-0.0012	0.0353	-0.1358	0.1338	0.998	0.998	-0.296	1.000

2) *Conventional UIP test*

The upper panel of Table 2 shows the results of a conventional UIP test with the equation (7) specification, $s_{t+1} - s_t = \alpha + \beta (f_t - s_t) + u_t$ (Model I), in the OLS framework.⁴² The individual null hypothesis of $\beta = 1$ and the joint null hypothesis of $\alpha = 0, \beta = 1$ are rejected at conventional significance levels. Moreover, the slope coefficient estimates are all negative. These results are consistent with the previous empirical studies. The lower panel of Table 2 displays the results of a conventional UIP test with the equation (8) specification, $s_{t+1} = \alpha + \beta f_t + u_t$ (Model II). Contrary to the results from the equation (7) specification, we fail to reject the null hypothesis of $\beta = 1$ at the conventional significance levels for any major currency. In addition, we also fail to reject the joint null hypothesis of $\alpha = 0, \beta = 1$ for any currency. The R^2 , the Durbin-Watson (DW) statistics and the Ljung-Box Q-statistics seem to be reasonable. These results are also consistent with the previous empirical studies.

In the above two specifications, however, if we want to adopt the standard t-test and F-test, each variable used in the estimation should be stationary. Hence, we need to apply unit root tests to the differenced spot exchange rate ($s_{t+1} - s_t$), the forward premium ($f_t - s_t$), the spot exchange rate (s_t) and the forward exchange rate (f_t). We will display unit root test results for these variables and other variables in the following section. The differenced spot exchange rate and the forward premium appear to be stationary variables. However, the spot and the forward exchange rates appear to be nonstationary variables.

⁴² We also estimated UIP condition in a seemingly unrelated regression (SUR) framework. According to Bilson (1981), there are two different reasons why we use SUR instead of OLS. First, all of the exchange rates are denominated in terms of the U.S. dollar, so that economic shocks originated from the U.S. will influence all of the exchange rates. Second, there has been formal and informal policy coordination in the foreign exchange market among major countries. However, the results are not much different from those of OLS.

Table 2. Conventional UIP test

$$\text{Model I: } s_{t+1} - s_t = \alpha + \beta (f_t - s_t) + u_t$$

	α	β	t-statistics	F-statistics	R^2	DW	Q(8)	Q(16)
			$\beta=1$	$\alpha=0, \beta=1$				
Canada	0.003** (0.001)	-1.602** (0.624)	-4.169***	8.786***	0.030	2.217	8.783	24.052**
Japan	-0.010*** (0.003)	-2.642*** (0.890)	-4.092***	8.376***	0.040	1.953	8.349	21.543
Germany	-0.002 (0.003)	-0.661 (0.780)	-2.128**	2.268	0.003	2.036	5.681	20.800
United Kingdom	0.006** (0.003)	-2.623** (0.958)	-3.783***	7.289***	0.034	1.934	5.003	20.050

$$\text{Model II: } s_{t+1} = \alpha + \beta f_t + u_t$$

	α	β	t-statistics	F-statistics	R^2	DW	Q(8)	Q(16)
			$\beta=1$	$\alpha=0, \beta=1$				
Canada	0.004 (0.003)	0.978*** (0.014)	-1.518	1.241	0.957	2.086	5.473	26.659**
Japan	0.004 (0.039)	0.999*** (0.008)	-0.119	0.009	0.988	1.792	9.483	20.650
Germany	0.003 (0.008)	0.996*** (0.011)	-0.354	0.066	0.973	1.990	6.727	22.405*
United Kingdom	-0.014 (0.008)	0.974*** (0.015)	-1.680	1.536	0.951	1.771	7.193	19.511

- Note: 1) The number in parenthesis indicates standard errors.
 2) *, **, *** denote that the values are significant at the 10%, 5%, and 1% significance level, respectively.
 3) DW is the Durbin-Watson statistic. Q(8) and Q(16) are Ljung-Box Q-statistic testing for autocorrelation at lags 8 and 16.

3) Linear cointegration tests

In the methodology section, we assume that f_t and s_t are cointegrated with a cointegrating vector (1, -1) or, equivalently, the forward premium ($f_t - s_t$) is stationary.⁴³

⁴³ According to the survey of Engel (1996), the empirical results on the stationarity of the forward premium have been mixed. In all survey, even though there is a mean-reverting behavior in the forward premium, the order of integration is known to be significantly less than one and greater than zero, i.e., they are fractionally integrated.

Before we test for the cointegration between s_t and f_t , however, we need to determine the univariate time series properties of these series. We will investigate the number of unit roots in the spot and forward exchange rates using Dickey-Fuller (1979) and Phillips-Perron (1988) tests.^{44, 45} Numerous studies have confirmed that the spot and the forward exchange rates for major industrialized countries are difference stationary, i.e. they are integrated of order one.⁴⁶ According to Table 3, the data used in this paper also confirm this finding that both the spot and forward exchange rate have a unit root for all currencies, i.e., they are $I(1)$. In contrast, differenced spot and forward exchange rates, the forward premium and the forecast error for each major currency do not have a unit root, i.e., they are all $I(0)$.

Now, we test for a cointegration relationship between the spot and forward exchange rates for both the unrestricted model (UR) and the restricted model (R).

$$(UR) \quad f_t = \alpha + \beta \cdot s_t + \varepsilon_{t+1} \quad (48)$$

$$(R) \quad f_t - s_t = u_t \quad (49)$$

In equation (48), a cointegrating vector is not specified whereas, in equation (49), a cointegrating vector is specified to be (1, -1). In the case of the unrestricted regression model, we conduct two different linear cointegration tests, the traditional Engle-Granger two-step approach and Johansen's maximum likelihood ratio test. In the case of the restricted regression model, the linear cointegration test with a known cointegration vector (1, -1) is

⁴⁴ Additionally, we will test unit root tests for $s_t - s_{t-1}$, $f_t - f_{t-1}$, $f_t - s_t$ and $s_t - f_{t-1}$. The stationarity of $f_t - s_t$ indicates indirectly that spot and forward exchange rate is cointegrated with a cointegrating vector (1,-1). Phillips-Perron (1988) test using the RATS program automatically includes a constant term.

⁴⁵ On the other hand, Enders and Granger (1998) provided a test of null of no cointegration against the alternative threshold nonlinear cointegration using a unit root test in the two regime threshold autoregressive (TAR) and momentum-TAR (M-TAR) models.

⁴⁶ See Meese and Singleton(1982), Baillie and Bollerslev(1989), Mark(1990), Liu and Maddala(1992), Crowder(1994), Clarida and Taylor(1997).

Table 3. Unit root tests on exchange rates

			S_t	f_t	$S_t - S_{t-1}$	$f_t - f_{t-1}$	$f_t - S_t$	$S_t - f_{t-1}$
Canada	ADF test	τ	0.483	0.477	-15.968***	-15.899***	-3.294***	-15.474***
		τ_μ	-1.645	-1.681	-16.002***	-15.931***	-5.245***	-15.455***
		τ_τ	-1.703	-1.736	-15.964***	-15.894***	-5.417***	-15.442***
	PP test	τ_μ	-1.537	-1.579	-16.088***	-16.014***	-5.169***	-15.496***
		τ_τ	-1.584	-1.622	-16.048***	-15.975***	-5.387***	-15.483***
Japan	ADF test	τ	-0.992(1)	-0.991(1)	-13.864***	-13.911***	-2.082**	-13.437***
		τ_μ	-0.443(1)	-0.433(1)	-13.893***	-13.941***	-2.888**	-5.952(5)***
		τ_τ	-2.805(1)	-2.799(1)	-13.907***	-13.957***	-3.095	-6.041(5)***
	PP test	τ_μ	-0.542	-0.538	-13.907***	-13.957***	-3.150**	-13.454***
		τ_τ	-2.423	-2.416	-13.919***	-13.970***	-3.457**	-13.527***
Germany	ADF test	τ	-0.856	-0.852	-14.800***	-14.848***	-2.345**	-14.576***
		τ_μ	-0.981	-0.984	-9.221***	-9.225***	-2.543	-9.009(1)***
		τ_τ	-1.693	-1.691	-9.228***	-9.233***	-3.397*	-9.141(1)***
	PP test	τ_μ	-1.061	-1.064	-14.802***	-14.848***	-2.469	-14.579***
		τ_τ	-1.774	-1.770	-14.791***	-14.840***	-3.418*	-14.686***
United Kingdom	ADF test	τ	-0.829	-1.827	-13.584***	-13.556***	-2.507**	-13.188***
		τ_μ	-1.638	-1.633	-13.563***	-13.535***	-3.203**	-13.167***
		τ_τ	-1.746	-1.742	-13.532***	-13.504***	-3.294*	-13.194***
	PP test	τ_μ	-1.638	-1.633	-13.576***	-13.548***	-3.303**	-13.200***
		τ_τ	-1.896	-1.896	-13.545***	-13.517***	-3.413*	-13.181***

Note: 1) The regression equation that can be used to test for the presence of a unit root is given as:

$$\Delta y_t = \gamma y_{t-1} + \sum \beta_i \Delta y_{t-i} + \alpha \quad (1-1)$$

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \sum \beta_i \Delta y_{t-i} + \alpha \quad (1-2)$$

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_2 t + \sum \beta_i \Delta y_{t-i} + \alpha \quad (1-3)$$

2) The statistics labeled as τ , τ_μ , τ_τ are the corresponding statistics to use for equations (1-1), (1-2) and (1-3), respectively. Phillips-Perron test always include a constant term.

3) The number in the parenthesis of ADF indicates the optimally chosen lag length in terms of AIC, while it is fixed at 3 for the Phillips and Perron (PP test).

*, ** and *** denote that the values are significant at the 10%, 5%, and 1% levels,

4) respectively.

simply a unit root test on the forward premium ($f_t - s_t$). We found in Table 3 that the forward premium for each currency does not have a unit root.

Engle-Granger two-step procedure. We can use the two-step procedure provided by Engle and Granger (1987). In the first step, we pretest the variables for their order of integration. If all variables are integrated of the same order, then we proceed to the second step in which we estimate the long-run equilibrium relationship among these variables using OLS, save the residuals, and perform a Dickey-Fuller test or an augmented Dickey-Fuller test on these residuals.⁴⁷ In this case, we use the Engle and Yoo (1987) critical values instead of the Dickey-Fuller critical values because the residual sequences are created from a regression equation and hence different from the true error process.⁴⁸

From the results of the unit root tests on the exchange rates, we found that the spot and forward exchange rates for any major currency are $I(1)$. Next, we estimate the long-run equilibrium relationship between the spot and forward exchange rates, alternating each variable as the left-hand-side variable. After that, we perform the augmented Dickey-Fuller test on the residuals. The detailed Engle-Granger tests are reported in Table 4. According to Table 4, we reject the null hypothesis of no linear cointegration between the spot and forward exchange rates for the Canadian dollar and the British pound but fail to reject the null hypothesis for the Japanese yen and the German mark. As we will give a detailed explanation in the later section, the failure to reject the null hypothesis for the latter two currencies is closely related to the period of the German unification from 1990 to 1996. Hence, we

⁴⁷ In this case, no intercept term is included in the regression of differenced residuals on the lagged residual because the residuals have zero mean by the original regression equation including the intercept term.

⁴⁸ According to Baillie and Bollerslev (1989), in finite samples, the estimated residuals will appear more stationary than the true value of the residual, and the DF critical values will be numerically too small, leading to a rejection of a unit root in the estimated residuals, i.e. finding cointegration, too often.

reestimate the ADF t-statistic of the Japanese yen and the German mark excluding the data during the German unification period and find that the spot and forward exchange rates for these two currencies are cointegrated for the subsample period from July 1978 through December 1989. The detailed test results are given at the second row of each currency in Table 4. Therefore, we conclude that the spot and forward exchange rates for each currency are cointegrated. The slope estimates are all very close to unity. However, Engle and Granger's method has a weakness for testing the null hypothesis of a cointegrating vector with (1,-1) because standard t test are not appropriate for testing the hypothesis.

Table 4. Engle-Granger cointegration test

Currency	$s_t = \alpha + \beta f_t + \mu_t$			$f_t = \alpha + \beta s_t + \mu_t$		
	α	β	ADF	α	β	ADF
Canada	-0.002 (0.000)	1.002 (0.002)	-5.345(0)***	0.002 (0.000)	0.997 (0.001)	-5.358(0)***
Japan	-0.011 (0.003)	1.003 (0.001)	-3.005(0)	0.012 (0.003)	0.997 (0.001)	-3.004(0)
	-0.005 (0.005)	1.002 (0.001)	-3.789(2)**	0.006 (0.005)	0.998 (0.001)	-3.792(2)**
Germany	-0.003 (0.001)	1.007 (0.001)	-2.948(0)	0.003 (0.001)	0.993 (0.001)	-2.949(0)
	0.003 (0.001)	1.000 (0.001)	-4.541(0)***	-0.003 (0.001)	1.000 (0.001)	-4.542(0)***
United Kingdom	-0.002 (0.001)	1.000 (0.001)	-3.443(1)**	0.002 (0.001)	1.000 (0.001)	3.442(1)**

- Note: 1) The number in the parenthesis of ADF indicates the optimally chosen lag length in terms of AIC under the maximum lag length of 6.
 2) *, **, *** denote that the values are significant at the 10%, 5%, and 1% levels, respectively.
 3) The Engle and Yoo (1987) critical values for two variables at the 1%, 5%, and 10% significance levels are -3.03, -3.37, and -4.07.
 4) The number in the parenthesis indicates standard errors.
 5) The second row of the Japanese yen and the German mark denotes the reestimated value using the subsample data from July 1978 to December 1989.

Johansen cointegration test. We can use the likelihood ratio test introduced by Johansen (1988) for testing cointegration relationships between spot and forward exchange rates. In contrast to the Engle and Granger two-step approach, the Johansen method makes it possible to test parameter restrictions.⁴⁹ The general multivariate Johansen procedure is related to the identification of the rank of the matrix Π in the model;

$$\Delta Y_t = A_0 - \Pi Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t \quad (50)$$

where Y_t and ε_t are n by 1 column vectors. The rank of Π equals the number of cointegration vectors. Johansen shows that Π can be written as the product of two n by k matrices of rank k , i.e., $\Pi = \beta \delta'$. In this paper, β and δ are 2 by 1 matrices of rank 1 if the spot and forward exchange rates are cointegrated. Hence, equation (49) can be rewritten in algebraic form as follows;

$$\begin{pmatrix} \Delta s_t \\ \Delta f_t \end{pmatrix} = \begin{pmatrix} \alpha_s \\ \alpha_f \end{pmatrix} + \begin{pmatrix} \beta_s \\ \beta_f \end{pmatrix} (\delta_f f_{t-1} - \delta_s s_{t-1}) + \begin{pmatrix} \sum_{i=1}^{p-1} \phi_n \Delta s_{t-i} + \sum_{i=1}^{p-1} \phi_f \Delta f_{t-i} \\ \sum_{i=1}^{p-1} \theta_n \Delta s_{t-i} + \sum_{i=1}^{p-1} \theta_f \Delta f_{t-i} \end{pmatrix} + \begin{pmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{pmatrix} \quad (51)$$

The Johansen maximum likelihood ratio test results are given in Table 5. From the maximum eigenvalue and trace statistics, we can conclude that the spot and forward exchange rates have one cointegrating vector for each currency. The likelihood ratio (LR) test for the null hypothesis of a cointegrating vector (1,-1) shows that we fail to reject the null hypothesis at conventional significance levels for any currency.

⁴⁹ The Johansen (1988) method has additional advantages over the Engle and Granger single equation approach. Unlike the Engle-Granger specification, the long-run coefficient estimates do not depend on the essentially arbitrary choice of the left-hand-side variable. Moreover this approach does not suffer from the small sample bias of the Engle-Granger static regression. Also, this method can estimate and test for the presence of multiple cointegration vectors. However, the Johansen procedure has been shown to be sensitive to the number of lags included in the VECM. We have to check whether the results are robust to variations in lag length.

Table 5. Johansen maximum likelihood ratio test

Country	Number of cointegrating vector				Test on cointegrating vector	
	$H_0: H_1$	λ_{trace}	$H_0: H_1$	λ_{max}	(δ_p, δ_r)	$H_0: \delta_i = -1.0$
Canada	$r=0, r>0$	24.83***	$r=0, r=1$	23.04***	(1.000, -0.995)	1.28
	$r \leq 1, r > 1$	1.80	$r=1, r=2$	1.80		
Japan	$r=0, r>0$	28.80***	$r=0, r=1$	27.41***	(1.000, -0.998)	1.55
	$r \leq 1, r > 1$	1.39	$r=1, r=2$	1.39		
Germany	$r=0, r>0$	11.78	$r=0, r=1$	10.67*	(1.000, -0.993)	2.22
	$r \leq 1, r > 1$	2.71	$r=1, r=2$	2.71		
United Kingdom	$r=0, r>0$	23.37***	$r=0, r=1$	18.66***	(1.000, -1.000)	0.01
	$r \leq 1, r > 1$	4.71	$r=1, r=2$	4.71		

- Note: 1) $\lambda_{\text{trace}}(r) = -T \sum_{i=r+1}^n \ln(1 - \hat{\lambda}_i)$, $\lambda_{\text{max}}(r, r+1) = -T \ln(1 - \hat{\lambda}_{r+1})$ where $\hat{\lambda}_i$ are the estimated values of the characteristic roots obtained from the estimated π matrix, and T is the number of usable observations.
- 2) The number of lags to use in the VECM is assumed to be one. The above results are robust to variations in lag length included in the VECM.
- 3) The test statistic for testing restriction on the cointegrating vector is $T \sum_{i=1}^r [\ln(1 - \hat{\lambda}_i^r) - \ln(1 - \hat{\lambda}_i)]$ where $\hat{\lambda}_i^r$ and $\hat{\lambda}_i$ denote the ordered characteristic roots of the unrestricted and restricted models, respectively, and r is the number of cointegrating vectors. This test statistic follows a χ^2 distribution with a degree of freedom equal to the number of restrictions placed on δ .
- 4) An element of the cointegrating vector, δ_j , is normalized to unity.

Cointegration test for restricted model. In the restricted model, we take an augmented Dickey-Fuller test and use their table for the critical value of pseudo t-statistic because the cointegrating vector is already known and used to obtain the true disturbance term. The unit root test result for the forward premium is already given in Table 3 and the null hypothesis of no linear cointegration is rejected.⁵⁰

⁵⁰ The interpretation of test results for linear cointegration is important to further research for threshold cointegration. Balke and Fomby showed that the conventional unit root tests such as the Dickey-Fuller test or the augmented Dickey-Fuller test used for linear cointegration are likely to be valid asymptotically for the threshold cointegration case. However, in finite samples, the stationary threshold processes is likely to behave locally as if they follow a unit root. So, traditional unit root tests may not uncover the existence of threshold cointegration. If we fail to reject the null hypothesis of no linear cointegration, we can continue to test nonlinear behavior of the forecast error ($s_{t+1} - f_t$) of exchange rate. In other words, even if we reject the null hypothesis of no linear cointegration, we still have the possibility to check whether the cointegration relationship is nonlinear.

4) *Threshold nonlinearity tests*

The fact that the forward premium is stationary, i.e. the spot and forward exchange rates are cointegrated with a cointegrating vector (1, -1) implies that equations (39) constitute a bivariate vector error correction model (VECM) in first differences of the spot and forward exchange rates including the error correction term. Equation (39) can be easily estimated by conventional OLS.⁵¹ The AR order p of the VECM was selected by the multivariate version of the Akaike Information Criteria (AIC) and the Schwartz Bayesian Criteria (SBC).⁵² The AIC and SBC choose $p=11$ and $p=0$, respectively, for the Canadian dollar, the Japanese yen, and the German mark and $p=1$ and $p=0$, respectively, for the British pound.⁵³ Based on these conflicting results, we select the parsimonious VECM with $p=0$ as the basic model for further studies. However, since autocorrelated errors due to the underspecification of AR order p may affect the nonlinearity test, we include VECMs with $p=1, 2, 3$ in our study to avoid the possible underspecification. The estimation results are given in Table 6. The slope estimates are all negative and inconsistent with the UIP condition of unity as we had explored earlier in this paper. Alternatively, we suggest a TVECM for capturing the UIP condition.

Before directly estimating the TVECM, we need to test whether equation (39) follows the multivariate threshold model against the alternative of a linear model. There are two

⁵¹ If each equation contains the same set of right-hand -side variables, OLS is an efficient estimation method. However, if each equation has different sets of right-hand-side variables, i.e. the different number of lagged variables, SUR is an efficient estimation method.

⁵² The multivariate generalizations of the AIC and SBC are given as follows;

$$AIC = T \log|\Sigma| + 2N, \quad SBC = T \log|\Sigma| + N \log(T)$$

where $|\Sigma|$ denotes the determinant of the variance/covariance matrix of the residuals and N =total number of parameters estimated in all equations.

⁵³ The AR order p can also be selected by the likelihood ratio (LR) test. The results in the case of our paper are almost the same as for the AIC.

different types of multivariate tests for threshold nonlinearity, Tsay's (1998) nonparametric method and Lo and Zivot's (2000) parametric method. Tsay's (1998) test is a generalization of his (1989) univariate version whereas Zivot and Lo's (2000) tests are extensions of Hansen's (1997, 1999) univariate test for detecting threshold nonlinearity. Tsay's (1998) test has good properties for empirical application. First, it is simple and performs well in finite

Table 6. VECM for (f_t, s_t) with a known cointegrating vector $(1, -1)$ ⁵⁴

$$\Delta s_t = \alpha_s + \beta_s(f_{t-1} - s_{t-1}) + \sum_{i=1}^P (\phi_{si} \Delta s_{t-i} + \phi_{fi} \Delta f_{t-i}) + \varepsilon_{st}$$

Country	P=0		P=1		P=2		P=3	
	α_s	β_s	α_s	β_s	α_s	β_s	α_s	β_s
Canada	0.003 (0.001)	-1.560 (0.623)	0.003 (0.001)	-1.338 (0.666)	0.002 (0.001)	-1.035 (0.699)	0.002 (0.001)	-1.078 (0.731)
Japan	-0.010 (0.003)	-2.601 (0.891)	-0.010 (0.004)	-2.628 (0.933)	-0.011 (0.004)	-2.901 (0.990)	-0.011 (0.004)	-3.033 (1.050)
German	-0.002 (0.002)	-0.745 (0.790)	-0.002 (0.003)	-0.588 (0.802)	-0.002 (0.003)	-0.490 (0.816)	-0.002 (0.003)	-0.526 (0.829)
United Kingdom	0.006 (0.003)	-2.578 (0.963)	0.006 (0.003)	-2.480 (1.016)	0.006 (0.003)	-2.248 (1.067)	0.006 (0.003)	-2.327 (1.109)

$$\Delta f_t = \alpha_f + \beta_f(f_{t-1} - s_{t-1}) + \sum_{i=1}^P (\theta_{si} \Delta s_{t-i} + \theta_{fi} \Delta f_{t-i}) + \varepsilon_{ft}$$

Country	P=0		P=1		P=2		P=3	
	α_f	β_f	α_f	β_f	α_f	β_f	α_f	β_f
Canada	0.003 (0.001)	-1.783 (0.625)	0.003 (0.001)	-1.539 (0.668)	0.002 (0.001)	-1.222 (0.701)	0.002 (0.001)	-1.270 (0.733)
Japan	-0.010 (0.003)	-2.685 (0.888)	-0.010 (0.004)	-2.734 (0.931)	-0.011 (0.004)	-3.005 (0.987)	-0.011 (0.004)	-3.121 (1.046)
German	-0.002 (0.002)	-0.805 (0.788)	-0.002 (0.003)	-0.655 (0.801)	-0.002 (0.003)	-0.546 (0.814)	-0.002 (0.003)	-0.575 (0.828)
United Kingdom	0.007 (0.003)	-2.669 (0.959)	0.006 (0.003)	-2.575 (1.011)	0.006 (0.003)	-2.337 (1.061)	0.006 (0.003)	-2.406 (1.104)

Note: 1) P is the number of AR order.
2) The number in the parenthesis is the standard error.

⁵⁴ We do not present the parameter estimates of lagged variables and the model specification test statistics such as R², Durbin-Watson (DW) statistic due to the space.

samples. Second, it is applicable for cointegrated systems and conditional heteroscedasticity. Finally, it does not depend on the form of the alternative threshold model and does not entail the problem of nuisance parameters.⁵⁵ However, Tsay's (1998) test has a weakness since it does not specify the form of the alternative threshold model. Once threshold nonlinearity is observed by Tsay's (1998) test, there is no explicit method to select a specific threshold model. In contrast, Lo and Zivot's (2000) has the advantage of specifying the form of the alternative threshold model but encounters the problem of nuisance parameters. Hence, the asymptotic distribution of Lo and Zivot's (2000) sup-LR (Likelihood Ratio) test statistic is dependent on the nuisance parameters and the bootstrap procedure has to be used to compute p-values of the test statistic. According to Lo and Zivot (2000), Tsay's (1998) multivariate test has higher power than the sup-LR test statistic to capture threshold nonlinearity since the non-parametric Tsay's test may have less model specification error. In this paper, we will apply both Tsay's (1998) and Lo and Zivot's (2000) multivariate tests for threshold nonlinearity.

Let's consider Tsay's test in the context of this paper. The basic model for testing threshold nonlinearity is given in equation (39). The threshold variable is assumed to be the lagged error correction term ($f_{t-d} - s_{t-d}$), i.e., the lagged forward premium. Tsay's (1998) multivariate threshold nonlinear test consists of several steps similar to his (1989) univariate test. First, a tentative AR order p and a set (S) of possible delay parameters d are selected. In the context of this paper, $p=0, 1, 2, 3$ are selected as we explained earlier. As for the possible delay parameter, we begin with $S=\{1, 2, 3, 4, 5, 6\}$ assuming that policy makers intervene in

⁵⁵ The problem of nuisance parameter means that the asymptotic distributions of the test statistics are dependent on the unidentified parameters (so called nuisance parameters) under the null hypothesis. In this case, the distribution of the test statistic is non-standard and thus, the bootstrap procedure is used to compute appropriate p-values.

the foreign exchange market within six months after a shock occurs in the market.⁵⁶ Second, we rearrange the sample data based on the increasing (or decreasing) order of the threshold variable and generate a sequence of recursive least squares estimates of the multivariate arranged regression for a given p and every element of S . Third, we obtain the standardized predictive residual of the multivariate arranged regression using a sequence of recursive least squares estimates computed by the second procedure. Then, we regress the standardized predictive residual on the multivariate arranged regressors and get the slope estimates of the regressors. The null hypothesis to be tested is zeros of the slope estimates of the regressors and the test statistic is as follows;

$$C(d) = [n - h - m_0 - (kp + vq + 1)] \times (\ln|\Sigma_0| - \ln|\Sigma_1|) \quad (52)$$

where d =the delay parameter, n =the number of observations, $h=\max(p,q,d)$, m_0 =the starting point of the recursive least squares estimation, k =the number of endogenous variables, p =the AR order of the endogenous variables, v =the number of exogenous variables, q =the AR order of the exogenous variables, $|\Sigma_0|$ =the determinant of the matrix Σ_0 , $|\Sigma_1|$ = the determinant of the matrix Σ_1 , and

$$\Sigma_0 = \frac{1}{n - h - m_0} \sum_{t=m_0+1}^{n-h} \hat{\eta}_{t(l)+d} \hat{\eta}'_{t(l)+d}$$

and

$$\Sigma_1 = \frac{1}{n - h - m_0} \sum_{t=m_0+1}^{n-h} \hat{w}_{t(l)+d} \hat{w}'_{t(l)+d}$$

⁵⁶ In most papers about the TAR model, d is assumed to be less than or equal to p . However, we don't need to keep this assumption.

where $\hat{\eta}_t$ = the standardized predictive residual and \hat{w}_t = the least squares residual of regression of the standardized predictive residual on the multivariate arranged regressors. Under the null hypothesis of linearity and some regularity conditions, Tsay's (1998) test statistic follows asymptotically a chi-squared distribution. Tsay's test statistics for the four major currencies are presented in Table 7. We find evidence that the VECM representation of spot and forward exchange rates for each currency has threshold nonlinearity. However, the results heavily depend on the choice of the delay parameter d and the AR order p . Additionally, note that finding evidence of threshold nonlinearity through Tsay's (1998) test does not provide any idea of the number of regimes.

Table 7. Tsay's multivariate threshold nonlinearity test

Country	Threshold Variables		d=1	d=2	d=3	d=4	d=5	d=6
Canada	$Z_{t,d} = (f_{t,d} - s_{t,d})$	P=0	2.08	6.24	7.48	12.79**	15.74***	15.67***
		P=1	15.05*	12.65	8.04	16.50**	17.92**	20.86***
		P=2	18.44	14.59	9.48	19.27*	18.47	20.59*
		P=3	29.78**	18.39	17.53	25.16*	23.62	30.70**
Japan	$Z_{t,d} = (f_{t,d} - s_{t,d})$	P=0	7.30	6.35	7.29	9.58**	9.72**	7.22
		P=1	16.50**	12.42	13.06	26.15***	18.36**	16.33**
		P=2	18.79*	14.56	17.47	27.05***	24.77**	21.31**
		P=3	21.89	16.27	19.72	35.34***	30.41**	20.04
Germany	$Z_{t,d} = (f_{t,d} - s_{t,d})$	P=0	7.07	7.55	0.32	3.15	3.40	2.0
		P=1	15.95**	16.80**	10.54	5.83	14.79*	23.73***
		P=2	27.22***	20.40*	14.68	16.90	17.51	25.13**
		P=3	32.44***	22.24	21.62	19.29	18.37	27.95**
United Kingdom	$Z_{t,d} = (f_{t,d} - s_{t,d})$	P=0	14.71***	4.50	4.10	2.88	2.18	9.47*
		P=1	27.01***	16.42**	3.85	7.46	7.03	8.64
		P=2	31.85***	17.95	5.03	12.46	13.35	13.20
		P=3	37.18***	25.60*	9.13	19.73	19.65	16.35

Note: 1) p is the number of AR order and d is the delay parameter.
 2) *, **, and *** implies that the statistic is significant at 10%, 5%, and 1% level, respectively.

In contrast, Lo and Zivot's (2000) test specifies the form of threshold nonlinearity and tests a null hypothesis of a linear VECM against the alternative of an m-regime TVECM using a sup-LR statistic.⁵⁷ However, since the threshold values and delay parameters are not identified under the null hypothesis, the distribution of the sup-LR statistic is nonstandard and should be obtained through a bootstrap approximation following Hansen (1997). In Lo and Zivot's (2000) test, we must estimate the m-regime TVECM to test for threshold nonlinearity. In this section, we present only the results of the threshold nonlinearity test,

Table 8. Lo and Zivot's multivariate threshold nonlinearity test

Country	Threshold Variables		Sup-LR ₁₂ test statistic	p-values	Sup-LR ₁₃ test statistic	p-values
Canada	$Z_{r,d} =$ $(f_{r,d} - s_{r,d})$	P=0	21.55(d=6)**	0.0210	47.64(d=6)***	0.0005
		P=1	30.94(d=6)**	0.0225	93.18(d=6)***	0.0000
		P=2	77.53(d=6)***	0.0000	105.45(d=6)***	0.0000
		P=3	84.11(d=6)***	0.0000	121.4(d=6)***	0.0000
Japan	$Z_{r,d} =$ $(f_{r,d} - s_{r,d})$	P=0	18.03(d=1)*	0.0755	22.66(d=1)	0.3110
		P=1	22.61(d=1)	0.1630	40.30(d=1)	0.1365
		P=2	28.17(d=1)	0.2375	61.01(d=1)**	0.0365
		P=3	29.38(d=1)	0.4975	66.28(d=1)	0.1665
Germany	$Z_{r,d} =$ $(f_{r,d} - s_{r,d})$	P=0	25.41(d=1)***	0.0060	32.87(d=1)**	0.0275
		P=1	67.44(d=6)***	0.0000	93.36(d=6)***	0.0000
		P=2	45.53(d=6)***	0.0030	96.88(d=6)***	0.0000
		P=3	60.78(d=1)***	0.0005	131.8(d=1)***	0.0000
United Kingdom	$Z_{r,d} =$ $(f_{r,d} - s_{r,d})$	P=0	11.07(d=2)	0.5180	33.16(d=2)**	0.0295
		P=1	33.31(d=4)**	0.0135	44.56(d=4)**	0.0795
		P=2	40.92(d=4)**	0.0145	60.06(d=4)**	0.0545
		P=3	65.19(d=4)***	0.0005	95.59(d=4)***	0.0040

- Note: 1) Sup-LR₁₂ and Sup-LR₁₃ indicate the test statistic of a linear TVECM against alternative equilibrium TVECM and band TVECM, respectively.
 2) P-values are obtained by 2000 bootstrap simulations.
 3) *, **, and *** implies that the statistic is significant at 10%, 5%, and 1% level, respectively.
 4) P is the AR order and d is the delay parameter.

⁵⁷ Since there is no conventionally defined LR statistic due to the nuisance parameter, the test statistic takes the sup-LR form.

providing details of the estimation and testing procedures in the later section. The results are given in Table 8. In order to report the approximate p-values for sup-LR_{12} and sup-LR_{13} statistic, we use the bootstrap with 2000 simulation replications for each AR order p . The VECM of the Canadian dollar and the German mark have threshold nonlinearity features in both two and three regime models irrespective of the AR order. The VECM of the British pound has threshold nonlinearity characteristics in both the two and three regimes for all AR order p except $p=0$. The VECM of the Japanese yen has threshold nonlinearity properties in the two regime model with $p=0$ and in the three regime model with $p=2$, which is a very different result from the result obtained from our application of Tsay's (1998) test. From these test results, we conclude that the VECM of spot and forward exchange rates for major currencies display evidence of threshold nonlinearity, although the evidence is severely dependent on the choice of the AR order and the delay parameter.

5) Model specification test

Even though we reject the linearity of the basic model against the alternative of threshold nonlinearity, we still have many problems to cope with. Some of the most difficult problems in estimating threshold nonlinearity models are to find an accurate number of threshold regimes and appropriate nuisance parameters, i.e., threshold values and delay parameters. Tsay (1998) does not provide a formal way to find the number of threshold regimes and appropriate nuisance parameters. Instead, he suggests an informal way in which people divide the data into subgroups according to the empirical percentiles of threshold variables and use the test statistic to detect any model change within each subgroup.

However, this method has a weakness in finding accurate threshold values and a concrete numbers of regimes. In contrast, Lo and Zivot's (2000) Hansen-type test allows for a systematic model specification test since it uses a sequential conditional least squares method based on nested models. In this paper, we focus on three regimes and thus we do not bother with the model specification test.

To decide between the equilibrium TVECM and the band TVECM, Lo and Zivot (2000) provide a likelihood ratio (LR) statistic

$$LR_{23} = T(\ln(|\hat{\Sigma}_2(\hat{\gamma}, \hat{d})|) - \ln(|\hat{\Sigma}_3(\hat{\gamma}, \hat{d})|)) \quad (52)$$

where $|\hat{\Sigma}_2(\hat{\gamma}, \hat{d})|$ and $|\hat{\Sigma}_3(\hat{\gamma}, \hat{d})|$ denote the determinants of the estimated residual covariance

Table 9. Lo and Zivot's model specification test

Country	Threshold Variables		Sup-LR ₂₃ test statistic	p-values
Canada	Z_{t-d} $= (f_{t-d} - s_{t-d})$	P=0	26.10***	0.001
		P=1	62.24***	0.000
		P=2	27.92	0.117
		P=3	37.28*	0.092
Japan	Z_{t-d} $= (f_{t-d} - s_{t-d})$	P=0	4.63	0.890
		P=1	17.68	0.217
		P=2	32.84**	0.043
		P=3	36.95*	0.097
Germany	Z_{t-d} $= (f_{t-d} - s_{t-d})$	P=0	7.46	0.587
		P=1	25.92**	0.036
		P=2	51.35***	0.001
		P=3	70.98***	0.000
United Kingdom	Z_{t-d} $= (f_{t-d} - s_{t-d})$	P=0	22.08***	0.010
		P=1	11.25	0.711
		P=2	19.14	0.516
		P=3	30.46	0.265

Note: 1) Sup-LR₂₃ indicates the test statistic of two regime TVECM against three regime TVECM.
 2) p-values are obtained by 2000 bootstrap simulations.
 3) *, ** and *** imply that the statistic is significant at 10%, 5%, and 1% significance level.

matrices from the equilibrium and the band TVECM, respectively, and $\hat{\gamma}$ and \hat{d} denote the vector of threshold values and the delay parameter, respectively. The asymptotic distribution of LR_{23} does not follow a conventional χ^2 distribution and hence, a bootstrap method has to be used to calculate p-values. Table 9 gives the LR_{23} test statistic and p-values. It shows that the test statistic of the Canadian dollar is significant at the conventional significance level when $p=0, 1,$ and $3,$ which implies that the band TVECM is significantly different from the equilibrium TVECM. Similarly, the band TVECM of the German mark is significantly different from the equilibrium TVECM at the 5% significance level when $p=1, 2,$ and $3.$ For the Japanese yen, the band TVECM is significantly different from the equilibrium TVECM at the 5% significance level when $p=2$ and $3.$ As for the British pound, the band TVECM is significantly different from the equilibrium TVECM at the 1% significance level only when $p=0.$

6) Model estimation and evaluation

Once the specific TVECM is determined to be the most appropriate model for capturing threshold nonlinearity behavior of the spot and forward exchange rates, the next step is to estimate the model. We can estimate the TVECM by least squares or maximum likelihood estimation. In this paper, we will follow the sequential conditional multivariate least squares method, following Hansen (1997) and Lo and Zivot (2000). The least squares parameter estimators of the TVECM are derived from solving the minimization problem. Let us explain the estimation procedure in the case of a band TVECM. First, for given threshold

values and delay parameters, we can estimate the parameters by the multivariate least squares method giving the residual sum of squares such as

$$S_3(\gamma_1, \gamma_2, d) = \text{trace}(\hat{\Sigma}_3(\gamma_1, \gamma_2, d)) \quad (53)$$

where $\hat{\Sigma}_3(\gamma_1, \gamma_2, d)$ denotes the estimate of the residual covariance matrix of the multivariate least squares conditional on (γ_1, γ_2, d) , and γ_1 and γ_2 are the lower and the upper threshold values, respectively, and d is the delay parameter. Second, the least square estimates of (γ_1, γ_2, d) are derived as follows;

$$(\hat{\gamma}_1, \hat{\gamma}_2, \hat{d}) = \arg \min_{\gamma_1, \gamma_2, d} (S_3(\gamma_1, \gamma_2, d)) \quad (54)$$

The above minimization problem can be best solved by a grid search for each possible value of γ_1, γ_2, d .⁵⁸ Since bootstrap methods take a lot of time to get the test statistic for the estimate, we restrict the possible set of γ_1, γ_2, d to facilitate the estimation and inference procedures. The asymptotic theory suggests that we should restrict the threshold, (γ_1, γ_2) , so that as $n \rightarrow \infty$, $n_j/n \geq \tau$ for some $\tau > 0$ and $j=1,2,3$.⁵⁹ This restriction requires that all three regimes have at least $n\tau$ observations.

Based on the previous threshold nonlinearity and model specification test, we choose a band TVECM ($p=0, d=6$) for the Canadian dollar, a band TVECM ($p=2, d=1$) for the Japanese yen, a band TVECM ($p=1, d=6$) for the German mark, and a band TVECM ($p=0, d=2$) for the British pound as the best band TVECM for each currency. Under mild regularity conditions, Tsay (1998) shows that the sequential conditional multivariate least squares

⁵⁸ Hansen (1997) indicated that even though the possible set of γ_1, γ_2, d is a bit large, it would not take much time to finish the full grid search with the help of the development of high speed personal computers.

⁵⁹ In practice, τ is set between 0.05 and 0.15. In our paper, we set 0.10 as a basis and adjust the value by the unit of 0.01 to find whether the estimation result depends on this value.

estimates are strongly consistent and are asymptotically normally distributed independent of the threshold values and the delay parameter. However, since we have no distribution theory for the parameter estimates in the finite sample case, standard errors do not provide much information on either the inference or the hypothesis test of the parameter estimates. However, we still report standard errors for possible future research purposes.

The estimated band TVECM for each currency is shown in Table 10. According to the test result, the slope estimate of the error correction term, i.e., the forward premium, in regimes 1 and 3 of the band TVECM is still negative in the Canadian dollar but positive in the German mark and the Japanese yen. As for the British pound, the results are mixed in the sense that the slope estimate of regime 1 is negative and the slope estimate of regime 3 is positive. The threshold values and the number of observations in each regime for the Canadian dollar seem very reasonable. The lower and the upper threshold values have different signs and regime 2 has the most of the observations. In contrast, the other three currencies have the same sign of threshold values and either regime 1 or regime 3 has too many observations. The standard deviations of the parameter estimates for the Canadian dollar are pretty small compared to the other three currencies. The Japanese yen has large standard deviations of the estimates in regimes 1 and 3 and the German mark shows large standard deviations of the estimates in regime 3. In contrast, the British pound shows relatively small standard deviations of the estimates in each regime compared to the Japanese and German currencies but still has large standard deviations in regime 3. Based on these findings, we argue that the parameter estimates in each regime for the Japanese yen and the German mark are suspicious although the slope estimates of the forward premium in regimes 1 and 3 are positive for the Japanese yen and German mark.

Table 10. The band TVECM for (f_t, s_t) with a known cointegrating vector (1, -1)

Country	variable	Regime 1		Regime 2		Regime 3	
		Δs_t	Δf_t	Δs_t	Δf_t	Δs_t	Δf_t
Canada (p=0, d=6)	<i>constant</i>	0.007 (0.003)	0.007 (0.003)	0.0003 (0.002)	0.0003 (0.002)	0.008 (0.003)	0.009 (0.003)
	$(f_{t-1} - s_{t-1})$	-1.832 (1.617)	-2.558 (1.616)	-2.004 (1.001)	-2.131 (1.000)	-2.120 (0.959)	-2.360 (0.958)
	z_{t-d}	$z_{t-d} \leq -0.0002,$ $n_1=22$		$-0.0002 < z_{t-d} \leq 0.0019,$ $n_2=122$		$z_{t-d} > 0.0019,$ $n_3=66$	
Japan (p=2, d=1)	<i>constant</i>	0.012 (0.011)	0.011 (0.011)	-0.002 (0.102)	0.0002 (0.102)	-0.009 (0.004)	-0.009 (0.004)
	$(f_{t-1} - s_{t-1})$	0.136 (1.937)	-0.045 (1.934)	6.536 (36.783)	7.415 (36.710)	0.728 (2.555)	0.667 (2.550)
	Δs_{t-1}	-1.939 (2.766)	-2.002 (2.761)	12.739 (10.486)	11.82 (10.466)	-2.802 (4.113)	-2.975 (4.105)
	Δs_{t-2}	1.933 (2.766)	1.989 (2.761)	-13.052 (10.468)	-12.142 (10.447)	2.943 (4.126)	3.117 (4.118)
	Δf_{t-1}	-0.468 (2.805)	-0.693 (2.800)	41.646 (13.440)	41.013 (13.413)	-5.136 (3.896)	-5.076 (3.888)
	Δf_{t-2}	0.474 (2.803)	0.706 (2.797)	-41.871 (13.449)	-41.241 (13.422)	5.157 (3.899)	5.098 (3.891)
	z_{t-d}	$z_{t-d} \leq -0.0032,$ $n_1=79$		$-0.0032 < z_{t-d} \leq -0.0025,$ $n_2=30$		$z_{t-d} > -0.0025,$ $n_3=101$	
	<i>constant</i>	0.019 (0.012)	0.017 (0.012)	-0.046 (0.010)	-0.047 (0.010)	-0.001 (0.004)	-0.001 (0.004)
Germany (p=1, d=6)	$(f_{t-1} - s_{t-1})$	2.730 (2.898)	2.198 (2.888)	-12.971 (2.891)	-13.347 (2.881)	0.260 (1.230)	0.264 (1.255)
	Δs_{t-1}	9.214 (3.270)	8.864 (3.259)	-6.309 (4.757)	-6.413 (4.741)	-3.501 (5.986)	-3.729 (5.966)
	Δf_{t-1}	-9.511 (3.304)	-9.161 (3.293)	5.999 (4.713)	6.102 (4.697)	3.704 (6.013)	3.928 (5.993)
	z_{t-d}	$z_{t-d} \leq -0.0034,$ $n_1=58$		$-0.0034 < z_{t-d} \leq -0.0018,$ $n_2=64$		$z_{t-d} > -0.0018$ $n_3=88$	
United Kingdom (p=0, d=2)	<i>constant</i>	0.007 (0.003)	0.007 (0.003)	0.118 (0.032)	0.114 (0.032)	-0.022 (0.019)	-0.021 (0.019)
	$(f_{t-1} - s_{t-1})$	-3.186 (1.530)	-3.276 (1.525)	-40.635 (10.321)	-39.487 (10.289)	3.690 (3.895)	3.577 (3.882)
	z_{t-d}	$z_{t-d} \leq 0.0027,$ $n_1=127$		$0.0027 < z_{t-d} \leq 0.0034,$ $n_2=21$		$z_{t-d} > 0.0034,$ $n_3=62$	

Note: 1) P is the AR order and d is the delay parameter.
 2) z_{t-d} is the threshold variable.
 3) The left-hand and right-hand side value of the inequality sign in the threshold variable row of each regime means the lower and the upper threshold values.

There is a possibility of structural change during the sample period, which might cause a bias in the slope estimate of the forward premium. If we look at Figure 4, the one-month forward premium, we can see the forward premium for the German mark and the Japanese yen are highly autocorrelated from 1990 through 1996, which is exactly the same period of the German unification process.⁶⁰ In other words, during the period of the German unification, the spot and forward exchange rates for the two major currencies against the U.S. dollar deviated continuously from the long-run equilibrium and came back again to the long-run equilibrium in 1996. In order to confirm this conjecture, we need to reestimate the TVECM for these two currencies considering the structural change. There are two ways to deal with the structural change. First, we may exclude the sample data for the two currencies during the German unification period. However, in this approach we lose some important information contained in the excluded data. Second, we may replace the denomination currency by the German mark or the other currency instead of the U.S. dollar. In our paper, we adopt both approaches and find that the second idea does not solve the structural change problem. Hence, we develop the first idea. We reestimated the TVECMs for the Japanese yen and the German mark using the subsample data for the period of July 1978 through December 1989. Table 11 contains the estimation results.^{61,62}

⁶⁰ With the fall of the Berlin wall in November 1989, the process of the German unification had been rapidly progressed. In May 1990, only a year later, the "State Treaty on the Economic, Monetary and Social Union" of two Germanys was signed and went into effect less than two months later in July 1990. Finally, in September 1990, the "Treaty on the Final Settlement with Respect to Germany" was signed in Moscow and thus Germany's unity and full sovereignty were restored. However, the German economy had stumbled for a few years since the unification and finally went back to the normal levels in 1997.

⁶¹ We also tested another possible structural change during the subsample period from October 1982 through July 1993. The selection of this subsample period was based on the timing of the major changes in U.S. monetary policy. According to M. Christensen (2000), U.S. monetary policy was changed from controlling money to controlling interest rates in October 1982 and the Federal Reserve decided not to use monetary targets any longer as a guideline for its monetary policy in July 1993. The test result does not provide better results than those from the entire sample period.

The newly estimated TVECMs for the two currencies are much different from the previous ones. Especially, the parameter estimates have very small standard deviations except those for the differenced lagged variables of regime 1 in the Japanese yen. The slope estimate of the forward premium term for each currency is still inconsistent with the UIP condition.

Table 11. The band TVECM for the Japanese yen and the German mark

Country	variable	Regime 1		Regime 2		Regime 3	
		Δs_t	Δf_t	Δs_t	Δf_t	Δs_t	Δf_t
Japan (p=1, d=4)	<i>constant</i>	0.063 (0.027)	0.062 (0.027)	-0.111 (0.032)	-0.109 (0.032)	-0.019 (0.006)	-0.020 (0.006)
	$(f_{t-1} - s_{t-1})$	7.063 (3.604)	6.785 (3.587)	-16.179 (5.017)	-15.926 (4.994)	-5.751 (1.847)	-5.956 (1.839)
	Δs_{t-1}	3.145 (5.069)	3.071 (5.046)	-13.756 (9.093)	-13.256 (9.050)	-5.195 (3.692)	-5.417 (3.674)
	Δf_{t-1}	-3.221 (5.172)	-3.148 (5.148)	13.199 (9.111)	12.674 (9.068)	5.162 (3.685)	5.379 (3.668)
	z_{t-d}	$z_{t-d} \leq -0.0068,$ $n_1=21$		$-0.0068 < z_{t-d} \leq -0.0049,$ $n_2=14$		$z_{t-d} > -0.0049,$ $n_3=97$	
Germany (p=1, d=5)	<i>constant</i>	0.031 (0.012)	0.029 (0.012)	-0.066 (0.017)	-0.067 (0.017)	-0.045 (0.010)	-0.046 (0.010)
	$(f_{t-1} - s_{t-1})$	5.757 (2.841)	5.198 (2.829)	-21.246 (4.833)	-21.686 (4.814)	-13.348 (3.164)	-13.547 (3.151)
	Δs_{t-1}	10.730 (3.304)	10.328 (3.291)	9.908 (5.221)	9.752 (5.201)	-16.320 (5.594)	-16.418 (5.572)
	Δf_{t-1}	-11.022 (3.343)	-10.622 (3.330)	-10.017 (5.166)	-9.857 (5.146)	16.077 (5.577)	16.169 (5.555)
	z_{t-d}	$z_{t-d} \leq -0.0039,$ $n_1=46$		$-0.0039 < z_{t-d} \leq -0.0030,$ $n_2=29$		$z_{t-d} > -0.0030,$ $n_3=57$	

- Note:
- 1) Subsample period is from July 1978 through December 1988.
 - 2) P is the AR order and d is the delay parameter.
 - 3) z_{t-d} is the threshold variable.
 - 4) The minimum fraction (τ) of total observations in each regime for the Japanese yen and the German mark is 0.1 and 0.11, respectively.
 - 5) The left-hand and right-hand side value of the inequality sign in the threshold variable row of each regime means the lower and the upper threshold values.

⁶² We also changed the minimum fraction (τ) of total observations in each regime to figure out whether the slope estimate is influenced by a change of this ratio. We changed the value of τ from 0.05 to 0.15 by increments of 0.01. The results are not much different from the results for $\tau=0.1$. As the last extension, we considered the possibility of a different threshold variable, replacing the forward premium (f_{t-1}) by the forecast error ($s_{t-1}f_{t-1}$), which is a more stable time series. However, this idea also did not help us find a better result.

These features, however, do not imply that the TVECMs for these currencies are misspecified. It only means that it is difficult to give plausible explanations to these estimation results from the perspective of the UIP condition. Hence, we can argue that even though the estimated TVECM does not fully explain the UIP puzzle, they may be used as a predictor of short-term movements in exchange rates. In the next section, we will prove this argument using out-of-sample forecasting.

Now, let us give some graphical interpretations to the estimated TVECMs to provide better ideas of what we have done so far. Figure 6 illustrates how the estimated thresholds divide each forward premium series into three regimes. Figure 7 depicts a linear regression in each regime of the TVECM for the Canadian dollar and also includes a graph of a conventional UIP regression.⁶³ As shown in Figure 7, a band TVECM is obtained by separating the original data cluster into three groups based on threshold values and then, by estimating a linear regression within each group.

Finally, we need to check the stability condition for the estimated TVECM. As presented earlier in this chapter, the stability condition is $|1 - c_1^j + c_2^j| < 1$, where c_1^j and c_2^j are the coefficients of the forward premium in regime $j=1, 3$. From Table 10 and 11, we have found that the TVECM for each currency satisfies this condition.⁶⁴

⁶³ We omit the graphs for the other three currencies since they look like almost the same pattern as that for the Canadian dollar.

⁶⁴ When we estimate TVECM using the entire sample, the German mark slightly violates the stability condition for the TVECM in regime 3 in the sense that $|1 - c_1^j + c_2^j|$ for the currency is almost 1. This problem may be caused by the persistent deviation of the spot and forward exchange rate from the long-run equilibrium relationship during the period of the German unification process.

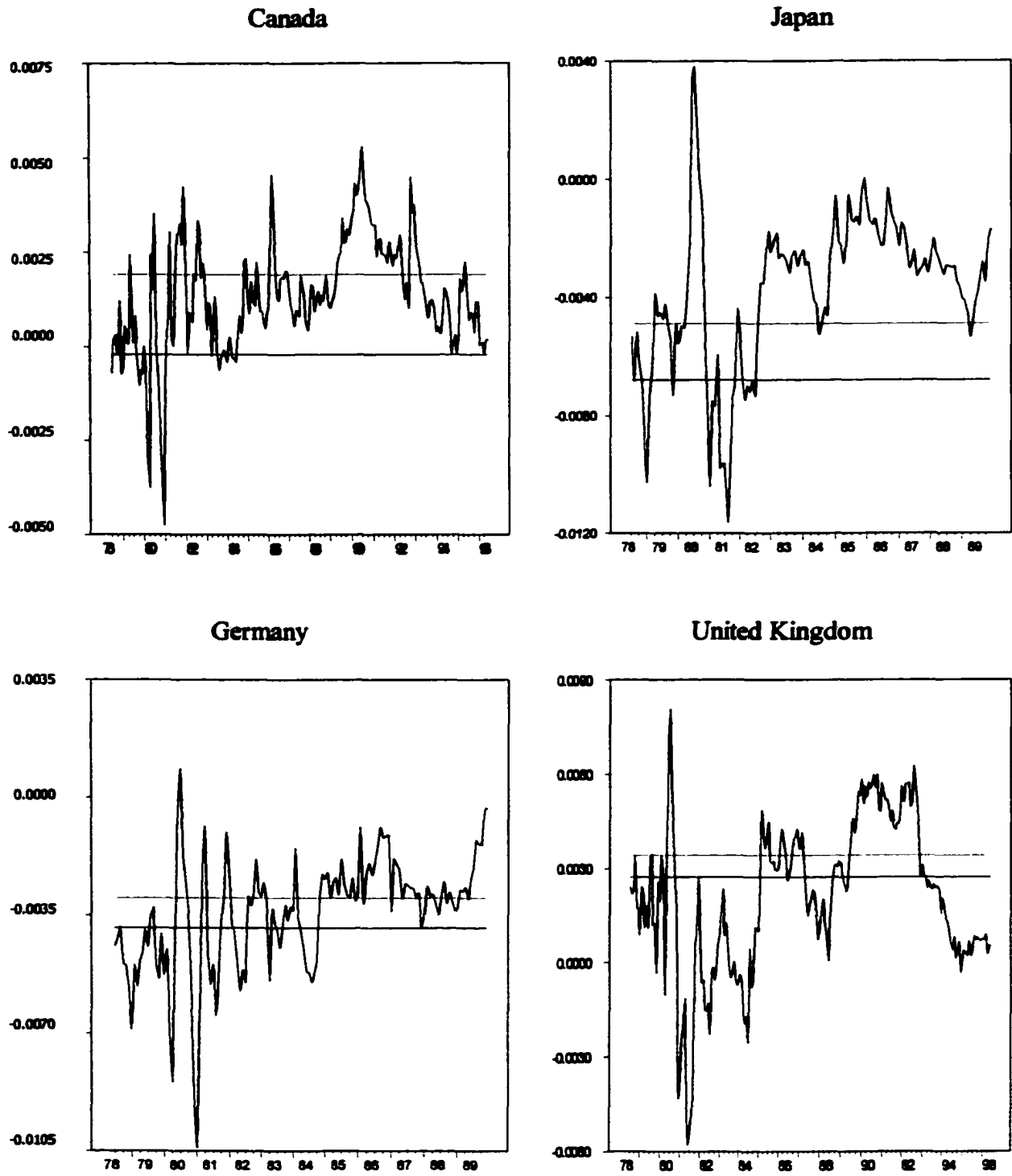


Figure 6. Three regimes of the band TVECM for four major currencies

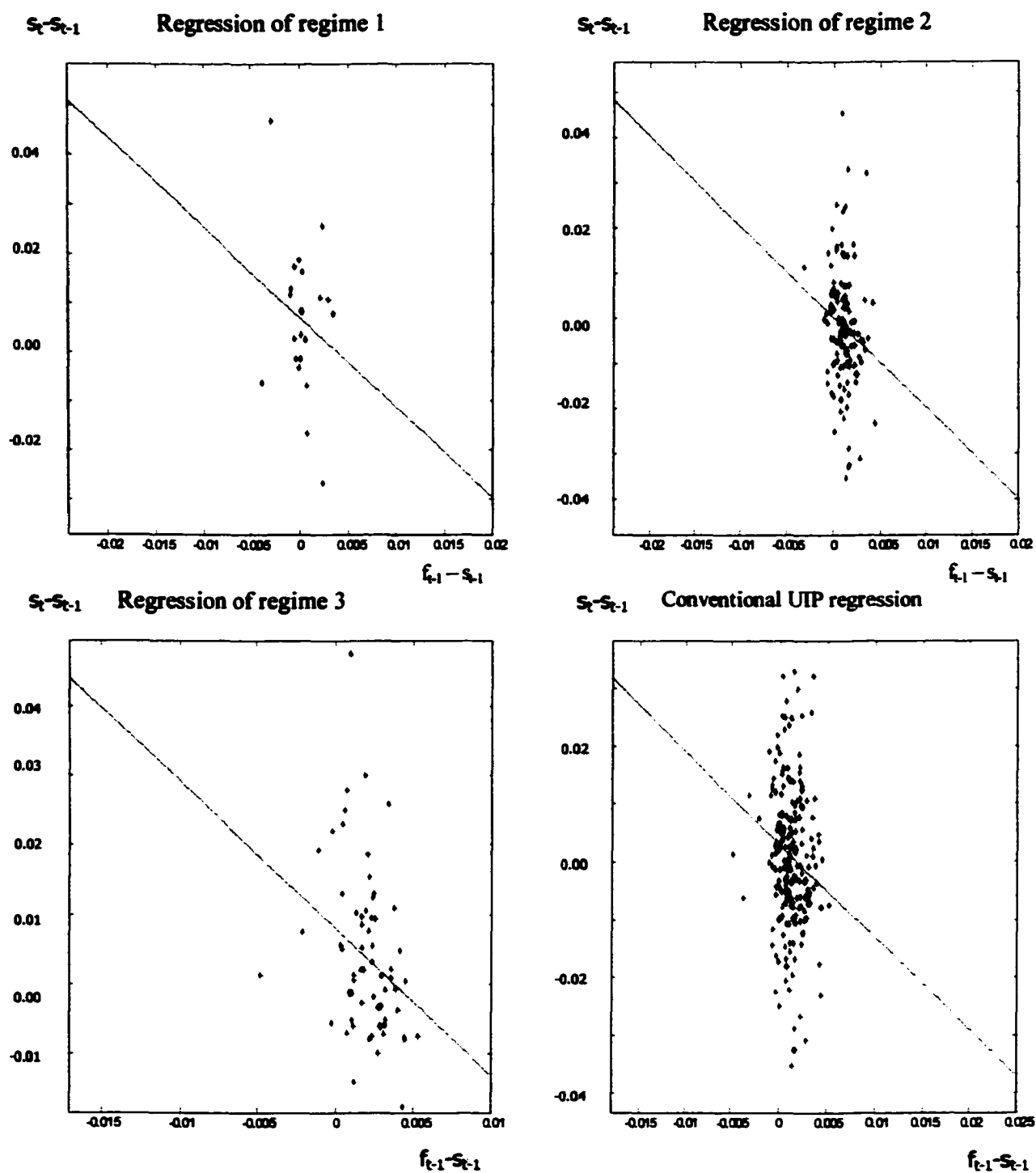


Figure 7. Regression in each regime of TVECM and conventional UIP regression (Canada)

We can summarize the estimation results as follows. First, we found positive slope estimates of the forward premium in regime 1 of some currencies using the threshold cointegration approach to solve the UIP puzzle. However, we must reluctantly admit that we did not succeed in finding robust evidence for the UIP condition. Second, we tested the possible variation of the original model such as a structural change in the sample period, changes in the minimum fraction (τ) of total observations in each regime, and the selection of an alternative threshold variable. These variations, however, did not provide a consistent result across the currencies. Instead, we found that the spot and forward exchange rates for the German mark and the Japanese yen deviated persistently from the long-run equilibrium relationship during the German unification period from 1990 through 1996, and that the TVECM for the two currencies should probably be re-estimated excluding the sample data for the period of the German unification.

7) Out-of-sample forecasting

Finally, we conduct an out-of-sample forecasting exercise using the estimated TVECM for the four major currencies. There are two purposes for the out-of-sample forecasting using the TVECM. First, the out-of-sample forecasting can be used to indirectly assess how well the TVECM is estimated. Second, through the out-of-sample forecasting, we can prove the argument that the TVECM may be used as a predictor of short-term movements in exchange rates.

In testing the out-of-sample forecasting power, we use rolling regressions with the parameters successively reestimated with each new data point. We estimate the out-of-

sample forecasts of the spot exchange rate one-month ahead for the four major currencies using the TVECM. The sample period for the Canadian dollar and the British pound is from July 1978 through June 1996 and the last twelve months from July 1995 through June 1996 are reserved for the out-of-sample forecasting test. In contrast, the sample period for the Japanese yen and the German mark runs from July 1978 through December 1989 and the last twelve months from January 1989 through December 1989 are reserved for the out-of-sample

Table 12. Comparison of forecasting results

		TVECM	VECM	Random Walk	Conventional UIP regression	
					Equation (7)	Equation (8)
Canada	RMSE	0.0081 (1.0000)	0.0089 (1.1008)	0.0083 (1.0181)	0.0086 (1.0625)	0.0084 (1.0350)
	MAE	0.0061 (1.0000)	0.0067 (1.0935)	0.0066 (1.0807)	0.00671 (1.0941)	0.0065 (1.0578)
Japan	RMSE	0.0347 (1.0000)	0.0364 (1.0509)	0.0367 (1.0598)	0.0375 (1.0826)	0.0391 (1.1280)
	MAE	0.0281 (1.0000)	0.0297 (1.0581)	0.0298 (1.0626)	0.0307 (1.0943)	0.0319 (1.1367)
Germany	RMSE	0.0342 (1.000)	0.0352 (1.0299)	0.0368 (1.0765)	0.0364 (1.0657)	0.0370 (1.0836)
	MAE	0.0279 (1.000)	0.0293 (1.0519)	0.0326 (1.1697)	0.0307 (1.1003)	0.0334 (1.1998)
United Kingdom	RMSE	0.0204 (1.0000)	0.0207 (1.0180)	0.0202 (0.9909)	0.0204 (1.0000)	0.0204 (1.0015)
	MAE	0.0171 (1.0000)	0.0175 (1.0218)	0.0166 (0.9698)	0.0171 (1.0011)	0.0168 (0.9812)

- Note: 1) Sample period for the Canadian dollar and the British pound is the whole sampling period from July 1978 through June 1996. In contrast, sample period for the Japanese yen and the German mark is from July 1978 through December 1989. Forecast period for the Canadian dollar and the British pound is from July 1995 through June 1996. In contrast, forecast period for the Japanese yen and the German mark is from January 1981 through December 1989.
- 2) The number in the parenthesis denotes its ratio to the corresponding figure for the TVECM. Thus a figure greater than 1 indicates superior relative performance by the TVECM.
- 3) RMSE and MAE denote root-mean-square-error and mean-absolute-error, respectively. TVECM and VECM denote threshold vector error correction model and vector error correction model, respectively. Forward premium regression and forward rate regression under the conventional UIP regression indicate equation (7) specification and equation (8) specification, respectively.

forecasting. Table 12 gives detailed results of the accuracy of these forecasts using the well-known root-mean-squared-error (RMSE) and the mean-absolute-error (MAE) criteria.⁶⁵ The table also gives the accuracy of four alternative forecasts: the forecasts produced by the first order VECM, a random walk forecasts, and forecasts produced by two conventional UIP regressions. In general, the TVECM outperforms the other models except for the random walk forecast of the British pound. For example, the RMSE of the TVECM forecast for the Canadian dollar is 0.0081 whereas the RMSE of the VECM forecast is 0.0089, which indicates a 10% reduction in RMSE by using the TVECM forecast against the VECM forecast. The TVECM forecast for the Canadian dollar against a random walk forecast shows about a 2% reduction in RMSE and an 8% reduction in MAE. These results are robust across the currencies. Even though we provide only one-month ahead forecasts for twelve months, the consistent superiority of the TVECM forecasts across the currencies does make us conclude that the TVECM for the four major currencies is well estimated, and that they can be used as a predictor of short-term movements of exchange rates.

⁶⁵ RMSE and MAE are defined as follows:

$$RMSE = \left[\sum_{j=0}^{N-1} (s_{t+j+h}^F - s_{t+j+h})^2 / N \right]^{1/2}, \quad MAE = \sum_{j=0}^{N-1} |s_{t+j+h}^F - s_{t+j+h}| / N \quad \text{where } h \text{ denotes the forecast step,}$$

s_{t+j+h}^F denotes the h month ahead forecast value of the spot exchange rate, s_{t+j+h} is the actual realized value of the spot exchange rate at time $t+j+h$ and N indicates the total number of forecasts in the out-of-sample period. In our paper, $h=1$ and $N=12$.

CHAPTER 5. CONCLUSION

In this paper, we have attempted to explain the puzzling results of the conventional empirical tests for the UIP condition using the threshold cointegration approach. Instead of taking a univariate threshold autoregressive (TAR) model used in the previous studies, we took a threshold vector error correction model (TVECM).⁶⁶ We argue that the negative slope estimate of the forward premium in the conventional single UIP equation might be caused by the misspecified model that fails to capture the nonlinear behavior of the data generating process. The estimation results using Lo and Zivot's (2000) Hansen-type multivariate test show that the slope estimate of the forward premium in the context of TVECM using the entire sample has positive or negative signs depending on the regime of the currencies. The Japanese yen and the German mark have positive slope estimates of the forward premium in the outer regime but the estimated coefficients are not close to the unity. In contrast, the Canadian dollar has still a negative slope estimate and the British pound has a positive slope estimate for the regime 1 and a negative slope estimate for the regime 3.

These mixed results for the four major currencies may be due to the innate structural changes in each currency during the sample period or the restriction of the minimum fraction of the observations in each regime or the possibility of an alternative threshold variable. Related to the structural changes, we found that the spot and forward exchange rates for the German mark and the Japanese yen against the U.S. dollar deviated continuously from the long-run equilibrium during the period of the German unification from 1990 through 1996. Hence, we reestimated the TVECM for the two currencies excluding the sample data during

⁶⁶ Strictly speaking, Coakley and Fuertes (1999) used the univariate threshold cointegration approach to test the forward rate unbiasedness hypothesis (FRUH) instead of the UIP condition.

the German unification period. The newly estimated TVECM for these currencies is found to have better statistical properties but is still inconsistent with the unity of slope estimate of the forward premium term suggested by the UIP condition. The Japanese yen, the German mark and the British pound have a positive slope estimate for regime 1 and a negative slope estimate for regime 3. However, the estimated coefficient of regime 1 for each currency is still not close to unity. In contrast, considering the other two possibilities such as changing the minimum fraction of the observations in each regime and the alternative threshold variable do not alter the main consequences of the initially estimated TVECM.

Finally, we constructed out-of-sample forecasts using the TVECM to assess the fitness of the estimated model and to confirm the argument that the TVECM may be used as a predictor of short-term movements in exchange rates. The out-of-sample forecast produced by using the TVECM is found to be generally superior to those obtained by using alternative models such as the VECM, a random walk model and the two conventional UIP regression models.

Based on these findings, we conclude that the threshold cointegration approach does not provide a robust evidence of the UIP condition, and that the UIP puzzle still remains partially unsolved. However, our paper gives some contribution to the study of the UIP puzzle and the application of the threshold cointegration approach. First, we took a general review of the theoretical and empirical studies on the UIP condition including the most recent research, especially the threshold cointegration approach. Second, we found that the spot and forward exchange rates for the four major currencies have the threshold nonlinearity. Third, we estimated the band and the equilibrium TVECM for the spot and forward exchange rates and found the results are partially consistent with the UIP condition. Forth, we constructed

out-of-sample forecasts using the TVECM and four alternative models and found that the TVECM has the best forecast ability based on the RMSE and MAE criteria. According to this finding, the estimated TVECM can be used as a predictor of short-term movements in exchange rates even though it does not fully explain the UIP puzzle.

As a topic of future study for the application of the threshold cointegration approach, we recommend the distribution property of the parameter estimate of the TVECM be founded. If we could get to understand the distribution property of the parameter estimate, we could make an inference or the hypothesis test on the parameter estimates of the TVECM.

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